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Probabilistic Analysis of Fatigue Life for ITER CS Conduit

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ABSTRACT

The fatigue life of ITER CS conduit has been designed based on deterministic fracture mechanics (DFM) combined with a safety factor. However, the method of safety factor does not quantify the failure probability of a mechanical system. Probabilistic fracture mechanics (PFM) analyzes data statistics, and estimates the fatigue life of a targeted mechanical system or components at selected failure probability or survival reliability.

Deterministic fracture mechanics (DFM) and the applied statistical methods are reviewed. A comprehensive statistical study of the ITER CS conduit has been performed for both Incoloy 908 and JK2LB based on existing test data and reasonable assumptions. The applied statistical methods include Monte Carlo simulation, small sampling statistics by student's t distribution, and uncertainty analysis.

The preliminary results indicate that a reduced stress may be necessary in order to obtain the required reliability for the fatigue life of ITER CS conduit. It is also found that both Incoloy 908 and JK2LB have similar fatigue life at 90% reliability, but that Incoloy 908 shows much better fatigue behavior at higher reliabilities. JK2LB has a significantly lower fracture toughness K_{1c}, and its limited database gives JK2LB a higher uncertainty in its estimated lifetime. The limited database for the Paris parameters of JK2LB greatly increases the uncertainty of its life due to small sample statistics. This indicates a need for more testing of JK2LB in order to improve the statistical estimates of its fatigue life.

1. INTRODUCTION

The traditional approach to conduit structural design is to apply deterministic fracture mechanics (DFM) multiplied by a safety factor.[1] Although simpler, this method does not quantify the probability of failure of a mechanical system. Probabilistic fracture mechanics (PFM)[2,3] analyzes data statistics, and then gives failure probability or survival reliability for a system or components.

Many probabilistic studies have been carried out over the last several decades in the pressure vessel and aircraft industries.[2,4,5] Some progress in the application of these methods has also been made for the ITER design project.[3,6] These previous ITER works use analytical methods to study the probability and uncertainty for maximum allowable stresses or minimum fatigue life at a given failure probability. In this report, a Monte Carlo method is applied. The Monte Carlo method has been extensively used in the statistical analysis of structural data,[4,7] and here it is further applied to uncertainty propagation. The Monte Carlo method gives accurate results, particularly for non-linear systems. Its main disadvantages are (a) a requirement for extensive CPU time; and (b) the contribution from each individual factor is not as apparent as by an analytical approach.

This report consists of 5 sections. The first section reviews the deterministic fracture mechanics (DFM), which is the backbone of all follow-up probabilistic studies, and the probabilistic fracture mechanics (PFM) is an extension of DFM into data statistics. The second section summarizes the statistical methods that are applied in the follow-up probabilistic analysis of fatigue life. The third and fourth sections report the statistical results, by the above methodology, for Incoloy 908 and JK2LB respectively. Finally a brief discussion includes a comparison between Incoloy 908 and JK2LB, as proposes some key conclusions.

2. REVIEW OF DETERMINISTIC FRACTURE MECHANICS

As the backbone of the follow-up statistical study, deterministic fracture mechanics (DFM) is reviewed first.[8-11] It includes fatigue crack growth (Paris law), load ratio effect, and fracture criteria.

The fatigue crack growth rate at constant stress amplitude is expressed by Paris law. It is only valid for linear-elastic fracture mechanics, but gives good approximation for the current simulation, in which a small scale yielding at the crack tip is assumed:

$$\frac{da}{dN} = C(\Delta K)^m \,, \tag{2.1}$$

where: stress intensity factor range $\Delta K = K_{\max} - K_{\min}$, C and m are Paris parameters, a and N are the crack size and fatigue cycle respectively. The expression of stress intensity factor K depends on crack configuration in addition to applied stress σ and crack size a:

$$K = Y\sigma\sqrt{\pi a} \,, \tag{2.2}$$

where: Y is the crack configuration factor, and a function of crack size, shape, location and specimen geometry. The specific formulations of Y for a 3D surface crack and embedded elliptical crack by Newman and Raju,[12] and Isida and Noguchi[13] are adopted in the current simulation.

Increasing mean stress $\frac{\sigma_{\max} + \sigma_{\min}}{2}$ for an applied stress range $(\sigma_{\max} - \sigma_{\min})$ generally shortens the fatigue life. The mean stress effect is accounted for by an effective stress intensity factor range:[14]

$$\Delta K_{eff} = K_{\max}(1-R)^{n} = \Delta K(1-R)^{n-1}, \tag{2.3}$$

where: stress ratio $R = K_{\min} / K_{\max}$, and n is the Walker exponent. Combining the above two equations gives:

$$\frac{da}{dN} = C(K_{\max})^{m}(1-R)^{mn}. \tag{2.4}$$

Fatigue life is estimated by the integration of Eq. 2.4 at constant stress amplitude by multi-point integration method.[11] Specifically, the integration is accomplished by a numerical simulation code at multiple peak K points (the points with maximum or minimum stress intensity factor K along crack periphery), i.e., 2 points for a surface crack (crack depth and crack surface edge), and 3 points for an embedded elliptical crack (2 points at each side in crack depth and 1 point at either side in crack length). A logic flow chart is shown in Fig. 2.1.

The effect of multiple stress amplitudes in CS coils is approximately calculated by either Miner's rule or multiple stress amplitude integration. Both approaches would give the same results as discussed in Ref. 9.

Failure criteria are defined by either the leaking of the conduit (i.e., crack depth exceeds the conduit thickness), or the final material fracture (i.e., maximum stress intensity factor exceeds the fracture toughness: $K_{\max} \geq K_{1c}$).

3. STATISTICAL METHODS

The main statistical methods applied hereafter are summarized in this section. They include Monte Carlo simulation, small sample statistics, and uncertainty analysis.

3.1 Monte Carlo simulation[4,7]

Monte Carlo simulation is used to analyze the uncertainty propagation from each input variable to fatigue life. The procedure consists of : (a) Fault tree diagram, which is a necessary preface to analyze the relation between system, component and various factors; (b) Random sample generation according to the distribution function of each input variable including crack configuration, material property, and applied load; (c) Fatigue life calculation and uncertainty propagation from each input variable to final output, i.e., fatigue life; (d) Probabilistic analysis of fatigue life including reliability estimation.

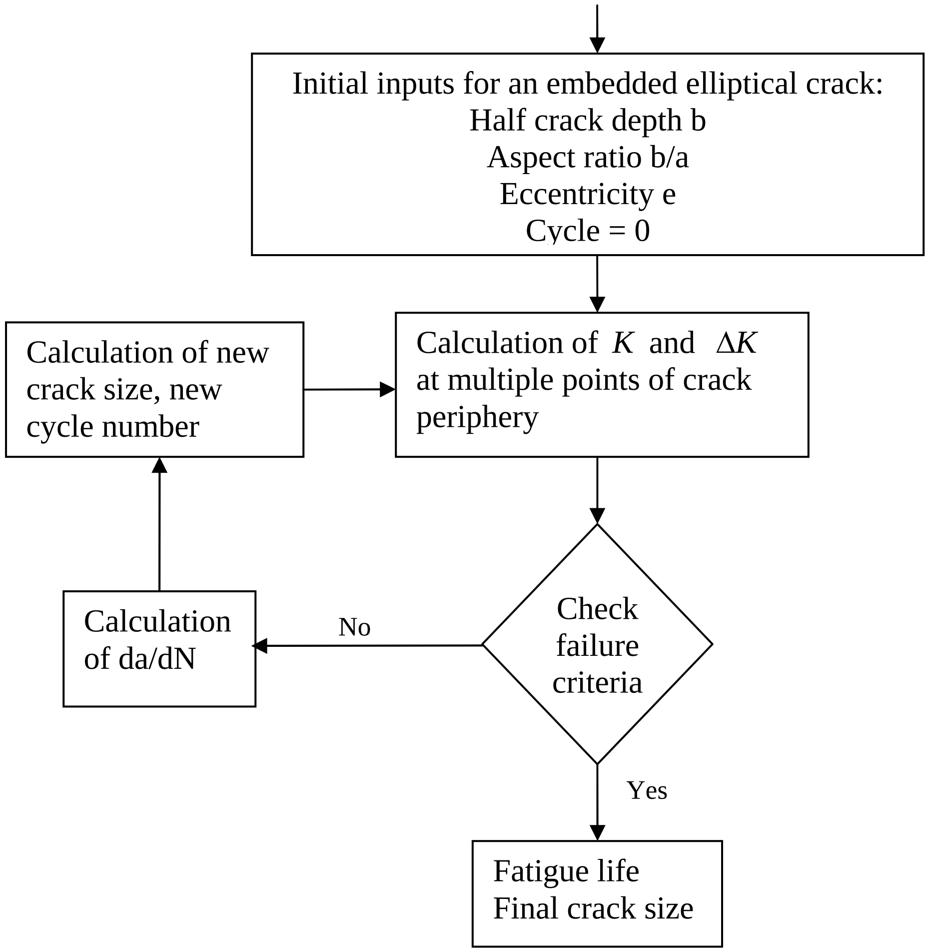

Fig. 2.1 Logic flow chart of fatigue crack propagation simulation

3.1.1 Fault tree diagram of failure mechanism[15]

The fault tree diagram for ITER CS conduit is shown in Fig. 3.1, which gives the relation among all variables for probabilistic fracture mechanics. The top links of each gate are output, and the bottom links are input. The gate "or" means that the output will exist if at least one input is present. The gate "and" means that the coexistence of all inputs is required to produce output. This diagram indicates that the input variables to eventually determine the failure Probabilistic of the CS system are the material properties (C, m, K_{1c}, n), the defect/crack distribution (size, shape and location), and the applied load (maximum stress and load ratio). The statistics of each variable will be discussed in the next section.

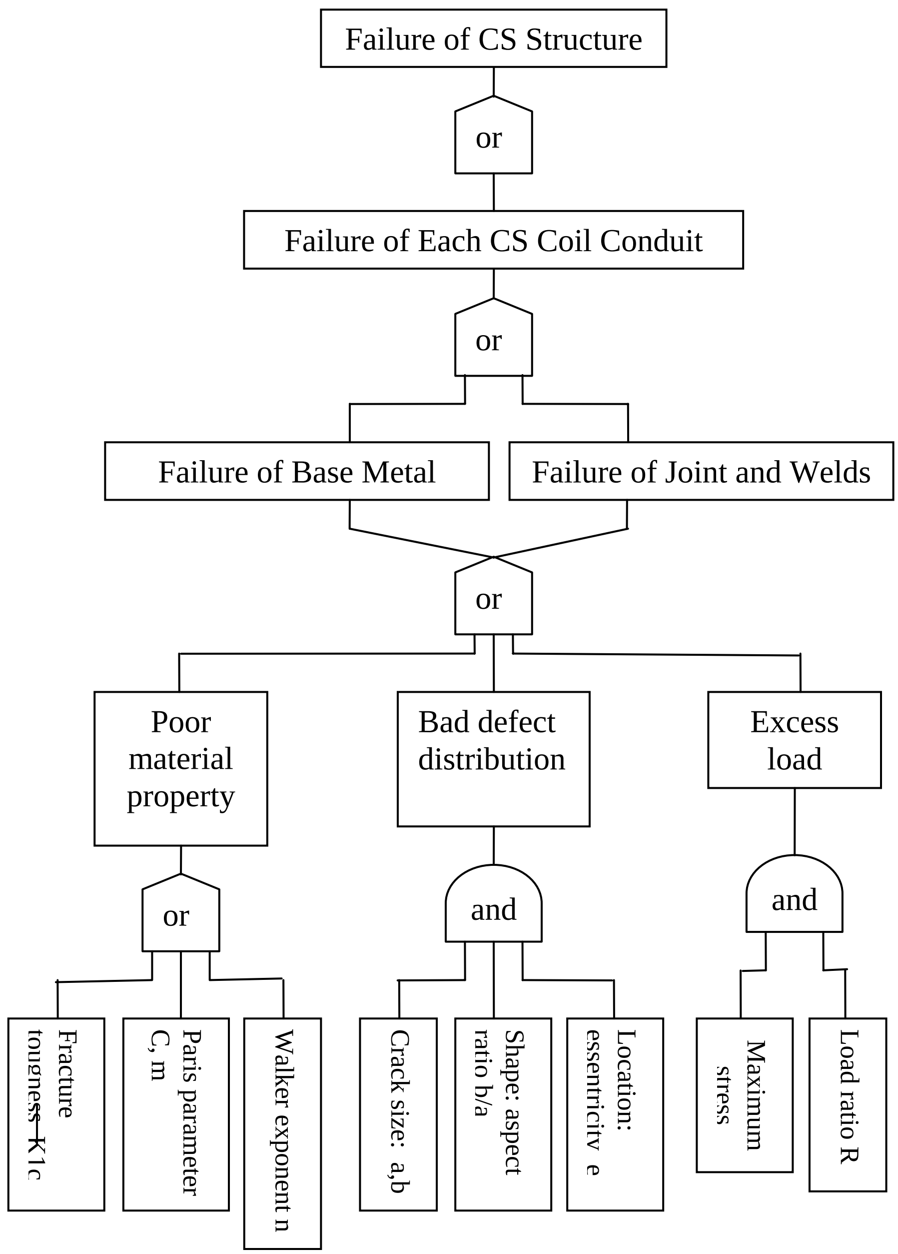

Fig. 3.1 Fault tree diagram of failure mechanism

3.1.2 Random sample generation of input variables [2,4,7]

The random sampling is typically performed in 2 steps: (a) generation of random number $r_j\,(j=1,2,3,\cdots n)$ by a random number generator; (b) using inversion method to get the random number $x_j\,(j=1,2,3,\cdots n)$ for given cumulative distribution function $F(x)$ as the following:[3]

$$x_j = F^{-1}(r_j), \quad j=1,2,3,\ \ldots,\ n. \tag{3.1}$$

The 2 steps can be combined into one by directly drawing random numbers from given distribution by using existing commercial codes, e.g. IMSL.

The specific random sampling distribution for each input variable is discussed in the next sections for a conduit made of Incoloy 908 and JK2LB respectively.

3.1.3 Fatigue life estimation and uncertainty propagation

The fatigue life estimation and uncertainty propagation from the input variables to the fatigue life is carried out by Monte Carlo simulation. The procedure consists of 4 steps, as shown in Fig. 3.2, the generation of random number, the random sampling of each input variable, the calculation of fatigue life for each set of random input variables, the statistical analysis for the sampling population of fatigue life.

Given each set of random input variables, the fatigue life can be calculated by using the procedure described in Sec. 2 and Fig. 2.1. The resulted fatigue lives form a sampling population for further statistical analysis.

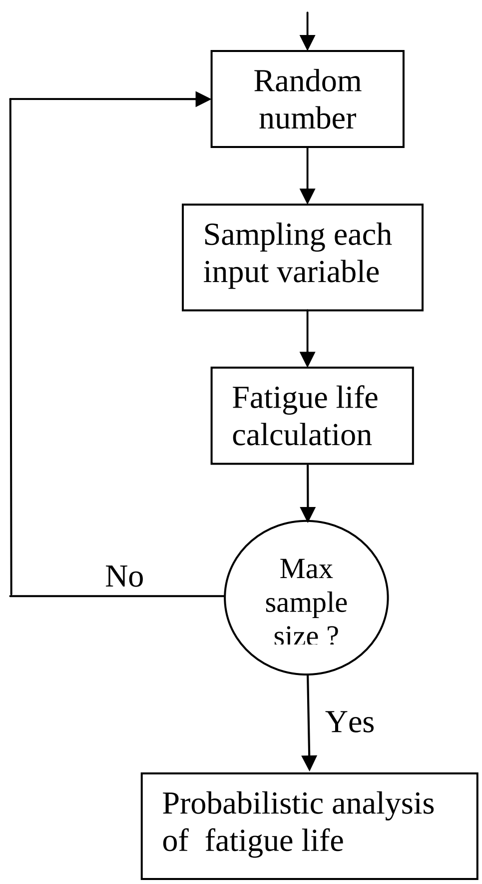

Fig. 3.2 Logic flow chart for Monte Carlo simulation

3.1.4 Probabilistic analysis of fatigue life

The probabilistic analysis of fatigue lives obtained from the Monte Carlo simulation can be performed either numerically or analytically. The numerical method includes: (a) Construction of probability density function $f(x)$; (b) Numerical integration of the density function into cumulative distribution function $F(x)$; (c) Construction of reliability function $R(x) = 1 - F(x)$. The analytical method [7,16] includes 3 steps: (a) Hypothesis of probability distribution; (b) Estimator of parameters; (c) Test of goodness of fit. The numerical method is especially good for large sampling population, e.g. the life data obtained by Monte Carlo simulation. The analytical method typically applies for a sampling population with limited data obtained by experiment. However, the analytical method is reviewed hereafter in order to help readers better understand both approaches.

3.1.4.1 Hypothesis of probability distribution

The Weibull distribution[7,16] has been most extensively used as a probability distribution function for the statistics of life test experiments. The others include exponential distribution, Gamma distribution, Normal distribution, and Lognormal distribution depending upon the specific life test data. The Weibull distribution was originally proposed by a Swedish scientist Weibull[16] for statistical analysis of material strength and then fatigue failure. It is adopted here as an alternative method to analyze statistically the fatigue crack growth life of CS conduits.

The probability density function of three-parameter Weibull distribution gives the probability of failure at ($x \to x + dx$), and is expressed as:

$$f\left(x \middle| \sigma, p, \gamma\right) = \frac{p}{\sigma}(x-\gamma)^{p-1} \exp\left\{-\frac{(x-\gamma)^p}{\sigma}\right\} , \tag{3.2}$$

where: x is a set of sample observations (i.e., the estimated fatigue lives) and x >0, p is a shape parameter, σ is a scale parameter, γ is a location parameter (virtually the threshold value of x). All the parameters are positive.

In the current life statistics, the threshold value of fatigue life is assumed to be zero, i.e., γ =0. The three-parameter Weibull distribution is therefore reduced to two-parameter Weibull distribution as:

$$f\left(x \middle| \sigma, p\right) = \frac{p}{\sigma} x^{p-1} \exp\left\{-\frac{x^p}{\sigma}\right\}. \tag{3.3}$$

Note that if p=1, the Weibull distribution reduces to the exponential distribution. According to the relations between the probability density function $f(x)$ and cumulative distribution function $F(x)$:

$$f(x) = \frac{dF(x)}{dx}, \quad \text{and} \quad F(x) = \int_{-\infty}^{x} f(x)dx \ , \tag{3.4}$$

we have the expression for the cumulative distribution function of two-parameter Weilbull distribution giving the probability of failure from 0 up to x:

$$F(x) = 1 - \exp\left\{-\frac{x^p}{\sigma}\right\} \ . \tag{3.5}$$

The probability of survival at x is expressed by the reliability function:

$$R(x) = 1 - F(x) = \exp\left\{-\frac{x^p}{\sigma}\right\} \ . \tag{3.6}$$

3.1.4.2 Estimator of distribution parameters

The next step would be estimation of the shape parameter p and scale parameter σ. The most common method is the Maximum Likelihood method. The Likelihood function is defined as:

$$L(x) = f(x_1) \cdot f(x_2) \cdot f(x_2) \cdots f(x_n) \ , \tag{3.7}$$

$$\ell n L(x) = \sum_1^n \ell n f(x_n) \ , \tag{3.8}$$

and for Weibull distribution

$$\ell n L(x) = \sum_1^n \ell n\left(\frac{p}{\sigma} x_i^{p-1} \exp\left\{-\frac{x_i^p}{\sigma}\right\}\right) = n\ell n \frac{p}{\sigma} + (p-1)\sum_1^n \ell n x_i - \frac{1}{\sigma}\sum_1^n x_i^p \ . \tag{3.9}$$

The principle of Maximum Likelihood method states that the estimates of unknown parameters are those values which maximize the Likelihood function $L(x)$. This is accomplished by taking derivative of Eq. 3.9 for p and σ, and we have:

$$n\sigma = \sum_1^n x_i^p \ , \tag{3.10}$$

$$\frac{\sum_1^n x_i^p \ell n x_i}{n\sigma} - \frac{1}{p} = \frac{\sum_1^n \ell n x_i}{n} \ . \tag{3.11}$$

Eqs. 3.10 and 3.11 can be solved by iterative method to obtain the distribution parameters p and σ.

A much simple and straight forward method using least square linear regression is proposed below specifically for Weilbull distribution. For the reliability function

$$R(x) = \exp\left\{-\frac{x^p}{\sigma}\right\} , \tag{3.6}$$

taking log in both sides of Eq. 3.6 and then rearranging the equation give

$$\sigma \ell n R(x) = -x^p , \tag{3.12}$$

and

$$\ell n \ell n R(x)^{-1} = p \ell n x - \ell n \sigma . \tag{3.13}$$

Plotting $\ell n \ell n R(x)^{-1}$ vs. $\ell n x$, and making a linear regression between them gives the required parameters p and σ.

3.1.4.2 Test of goodness of fit

The final step is to test the goodness of fit for the assumed distribution function and its parameters against the sample population. There are two classes of method. One is by the analytical, and another one is by Probabilistic Plot. The Probabilistic Plot uses the specific probability paper for given distribution to graphically display the test outputs. It is an easy and straight method. The disadvantage is that it needs specially formatted paper for probability plotting purpose.

A new graphical method is proposed below to achieve the same task, and to avoid using the specially formatted paper. For each sample observation range from ($x - dx$) to ($x + dx$) , we can calculate, from both the sample population and the estimated distribution, the distribution density or the cumulative function. Then the ones from the sample population are plotted against those from the estimated. If it fits well, the data should be displayed as a straight line in 45 degree against the both axes.

3.1.4.3 Reliability and failure odds

With the estimated distribution and its well fitted parameters in hand, we can easily obtain the reliability R(x) at given fatigue life as shown in Eq. 3.6:

$$R(x) = \exp\left\{-\frac{x^p}{\sigma}\right\}. \quad (3.6)$$

For given failure odds, the fatigue life can be obtained by the inversion:

$$x = \{-\sigma \ell n R\}^{1/p}. \quad (3.14)$$

3.2 Small sampling statistics [3, 17]

Assume that a variable *x*, e.g., *log(life)*, has a set of n data: $x_1, x_2, x_3, \cdots, x_n$. The mean of *x* is:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i . \quad (3.15)$$

The data dispersion is measured by standard deviation as:

$$s_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} . \quad (3.16)$$

It is found that the data distribution is approximately normal if the data number is greater than 30, and it becomes a perfect normal distribution if the data number is infinite. However, for most engineering problems, the data number is less than 30 and its distribution is deviated from a normal distribution. For such small sample problems, student's t distribution applies.

A student's t distribution is a modification of the normal distribution graphically. The flatness of a student's t distribution is a function of the degree of freedom (DOF) (e.g., DOF=n-1 for a single variable x with n data). Less data number leads to a flatter curve than the normal distribution. The confidence limits for the mean of student's t distribution are:

$$x_{conf}^{mean} = \bar{x} \pm t_{c,f}\, s_x \left(\frac{1}{n}\right)^{1/2}, \quad (3.17)$$

where $t_{c,f}$ is a critical value, a function of the confidence limits and DOF. As the data number n increases, the confidence limit of mean x_{conf}^{mean} approaches to the mean value $\bar{x}$. The prediction limits for one more observation are:

$$x_{conf}^{prediction} = \bar{x} \pm t_{c,f}\, s_x \left(1 + \frac{1}{n}\right)^{1/2}. \quad (3.18)$$

As the data number n increases to greater than 30, the confidence limit of one more prediction $x_{conf}^{prediction}$ approaches the normal distribution. On the contrary, a decreased data number n leads to a larger critical value $t_{c,f}$, and thus a larger deviation of one more observation from the mean at a given confidence limit.

3.3 Uncertainty analysis [3, 6]

Assume $F(x)$ is a linear function of independent variable x_i as

$$F(x) = F(x_1, x_2, x_3, \cdots) . \tag{3.19}$$

The center of the uncertainty interval for the function $F(x)$ is the function of the centers of the uncertainty interval for each independent variable x_i as:

$$F_0 = F(x_{01}, x_{02}, x_{03}, \cdots) . \tag{3.20}$$

Assume that an independent variable x_i is within a range by its central value x_{oi} plus or minus an uncertainty Δx_i at given odds:

$$x_{0i} - \Delta x_i < x_i < x_{0i} + \Delta x_i . \tag{3.21}$$

The uncertainty of the function $F(x)$ at given odds due to the uncertainty of x_i is:

$$u_i = F(x_{01}, x_{02}, \cdots, x_{0i} \pm \Delta x_i, \cdots) - F_0 . \tag{3.22}$$

The total uncertainty of the function $F(x)$ at given odds due to the uncertainties of all independent variables is estimated as:

$$u_{tot} = \sqrt{\sum u_i^2} . \tag{3.23}$$

The one more prediction for the function $F(x)$ at given odds is then:

$$F_{conf}^{prediction} = F_0 \pm u_{tot} . \tag{3.24}$$

If the function $F(x)$ is non-linear, the above approach is still valid approximately.

4. PROBABILISTIC ANALYSIS ON CS CONDUIT MADE OF INCOLOY 908

The Probabilistic analysis on conduit fatigue life described in the ITER Design Description Document DDD11[1] only takes consideration of the statistical distribution of crack size distribution, and is therefore incomplete. A more comprehensive study of the probabilistic fracture mechanics based on fatigue crack growth for ITER CS conduit made of Incoloy 908 has been performed, and the preliminary results are reported hereafter.

This analysis is based the principles discussed in above 2 sections, and listed in Table 4.0. It consists of 4 steps: (a) The major effects associated with crack configuration, material property, and load variables, it is carried out by a Monte Carlo simulation; (b) The uncertainty due to limited number of test specimen for fatigue crack growth, it is analyzed by small sample statistics; (c) The minor effects induced from many other minor factors. Finally, the reliability of CS conduit is estimated after taking consideration of the volume effect. In this study, the effect of stress riser around the joints is not included.

Table 4.0 Logical list of statistical analysis

Step No.	*Effect*			*Solution*
A	Major effect from fatigue crack growth	Crack configuration	Crack size	Monte Carlo simulation
			Crack shape	
			Crack location	
		Material property	Paris parameters	
			Fracture toughness	
			Walker coef.	
		Applied load	First peak stress	
			2^{nd} peak stress	
			Load ratio	
B	Uncertainty due to limited number of crack growth specimen			Small sample statistics by student's t distribution
C	Minor effects	Short crack		Previous work lore and assumptions
		Plastic zone		
		T stress / Z stress		
		Plate thickness		
		Load shedding		
		……………….		
D	Volume effect and uncertainty analysis			Eqs. 4.7 to 4.9

4.1 Major effect associated with fatigue crack growth

The major effect associated with fatigue crack growth is analyzed by Monte Carlo simulation. It includes 3 groups of variables (a) crack configuration: crack size, crack shape and crack location; (b) material property: Paris parameters, fracture toughness and Walker coef.; (c) load variables: the 1^{st} peak stress, the 2^{nd} peak stress and the residual stress. In the current Monte Carlo simulation, 3000 random sampling points are drawn from given distribution for each input variable. The fatigue life for each set of input random sampling data is calculated by the deterministic fracture mechanics using Paris law integration.

4.1.1 Input variables

4.1.1.1 Crack configuration

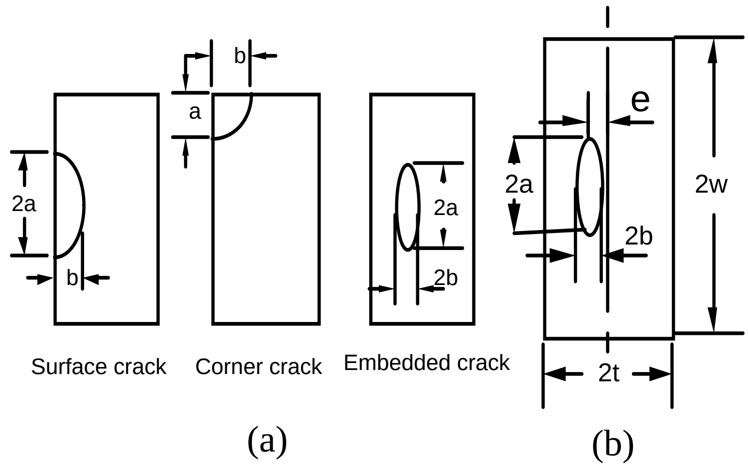

Fig. 4.1 Crack configuration (a) 3 type of cracks (b) definition of an embedded crack

A schematic representative of crack configuration is shown in Fig. 4.1, in which only the embedded crack and surface crack have been studied. For an elliptical embedded crack, “b” is defined as the half crack depth, “a” is the half crack length, crack aspect ratio is “b/a”, “e” is the eccentricity representing the crack location. A surface crack is equivalent to a subsurface elliptical crack with crack depth and aspect ratio doubled.

The defect size distribution in CS conduit is probably one of the most significant issues for probabilistic evaluation of fatigue life. Unfortunately, there is no data base for CS conduit made of either Incoloy 908 or JK2LB. In this study, we will use the published data from the pressure vessel and piping industry. Among many models[2] for the crack size distribution, the Marshall model[18] is perhaps the most typical one. It is expressed as an exponential form:

$$p(b) = \frac{1}{\mu}\exp\left(-\frac{b}{\mu}\right) , \tag{4.1}$$

where b is the crack depth and μ is the mean depth. The cumulative distribution of the crack depth is then obtained from the integration of $p(b)$ from 0 to b:

$$F(<b) = 1 - \exp\left(-\frac{b}{\mu}\right) . \tag{4.2}$$

By assuming 95% probability after inspection for a crack with area of 0.75 mm^2 and an aspect ratio of 0.5 (i.e. a half crack depth of 0.345 mm for an elliptical crack), we obtain the crack mean value $\mu = 0.266mm$. It says that we have 95% confidence that the crack size in the material does not exceed 0.346mm.

A set of random data (i.e., 3000 points) for crack depth based on the exponential distribution Eq. 4.1 are drawn and shown in Fig. 4.2, in which the crack depth is divided into 50 cells from the small to the large, and each cell size is 0.023 mm. The probability in y axis represents the percentage of frequency in each cell.

It is worthwhile to note that the assumption made in DDD11[1] to define crack area instead of crack depth as exponential distribution would result into non-exponential distribution of crack depth, and therefore contradicts all test data published in references. In addition, the true crack size distribution comes from the combined effect of initial crack distribution and subsequent detection probability during inspection, the assumption of 95% detection probability used frequently in DDD11 criteria does not addresses the initial crack distribution for existing cracks, and therefore is incomplete.

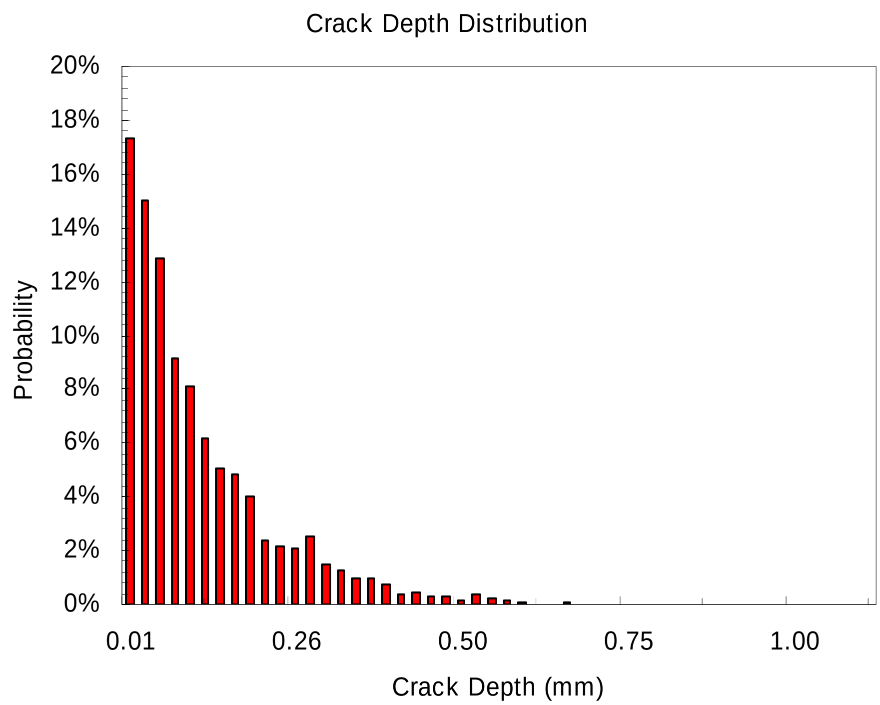

Fig. 4.2 Exponential probability distribution of the crack depth (e.g., half depth for an elliptical embedded crack and full depth for a surface crack). The Y axis represents the probability of crack depth in each cell, total 50 cells along x axis with cell size of 0.023 mm.

Crack aspect ratio $r = b/a$ is another important parameter of a 3D crack. However, very little information is available in this field. References 2 and 19 use the data from NDE test, and give a normal distribution of the aspect ratio with mean value $\bar{r} = 0.5$ and standard deviation $\sigma = 0.16$. 3000 random normal data for the aspect ratio are drawn, and shown in Fig. 4.3.

$$p(r = b/a) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(r-\bar{r})^2}{2\sigma^2}\right) \quad . \qquad (4.3)$$

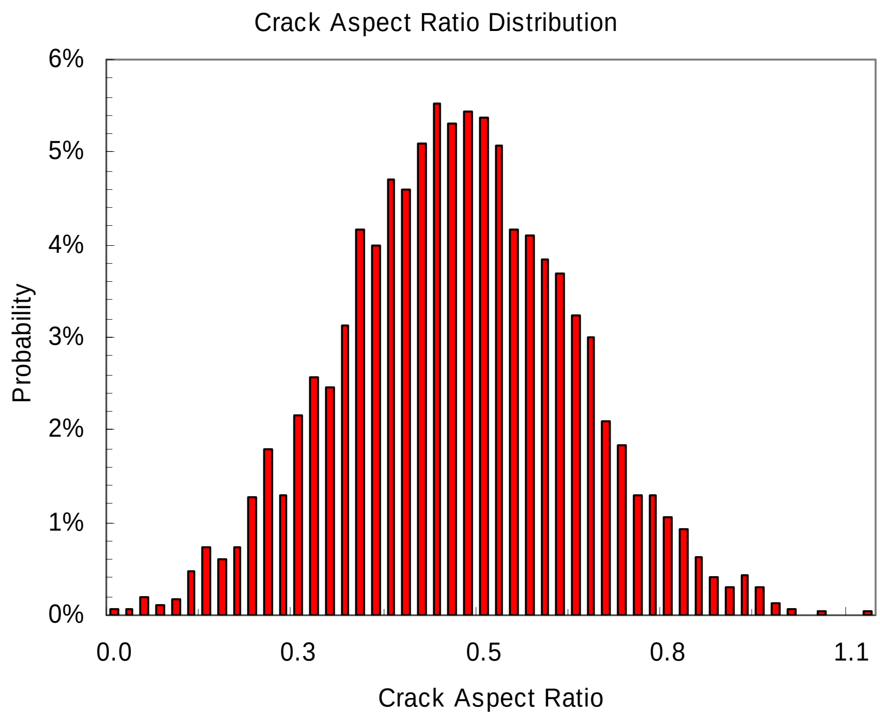

Fig. 4.3 Normal probability distribution for crack aspect ratio, 50 cells, cell size = 0.022.

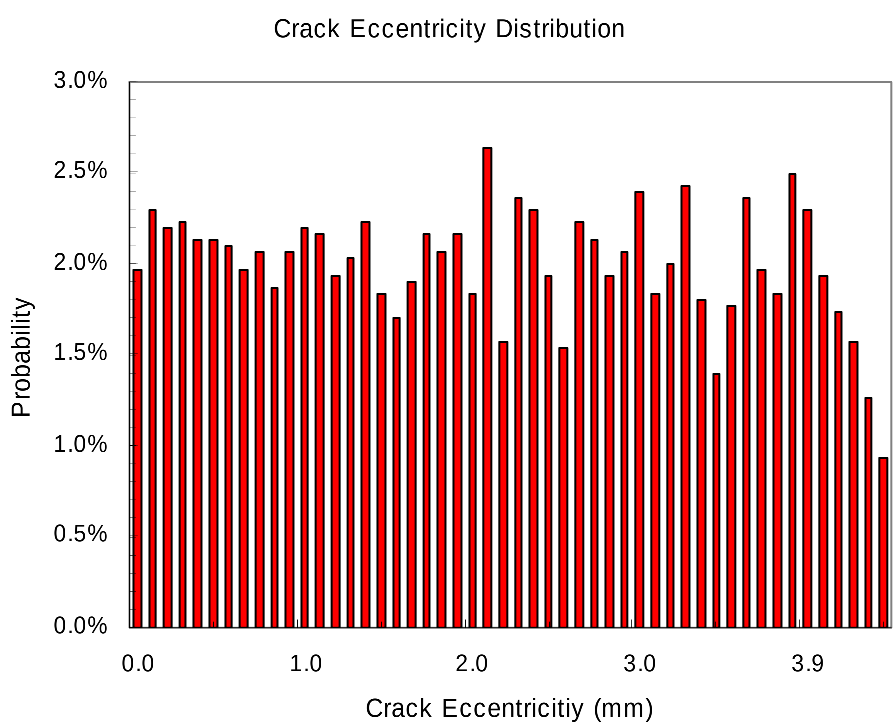

Fig. 4.4 Uniform distribution for crack eccentricity, 50 cells, cell size= 0.09 mm

The crack location distribution depends on material manufacturing procedure. There is no any published data available. Experience indicates that most cracks likely stay on surface or near the surface. However, a surface or near-surface crack is more easily found by a NDE test, and then removed. Therefore, we assume that the eccentricity of a 3D crack (i.e., the deviation of the crack center from the thickness center) follows a uniform distribution. Fig. 4.4 shows a set of random data for uniform distribution drawn from 3,000 random samples for the eccentricity of a 3D crack.

4.1.1.2 Material property

The material property for fatigue crack propagation includes Paris parameters (*C, m*), fracture toughness K_{1c}, and Walker exponent *n*.

Paris parameter (*C, m*) is the most important material property. It defines the crack growth resistance of this material. The pair of Paris parameter (*C, m*) is mutually related, and therefore can not be analyzed independently. We adopt hereby the approach in reference 2, in which *m* is assumed to be constant and let *C* to be an independent variable. It is found that *Log(C)* follows a normal distribution, and its 2 visible boundaries of data scattering represent 10% and 90% probability respectively. In the current analysis, the mean value of *Log(C)* is the listed value and the standard deviation is approximately 0.29 estimated by statistical analysis for the published crack growth data in Reference 7. 3000 random data of normal distribution for *Log(C)* are drawn, and shown in Fig. 4.5 for a typical Paris parameter *C* mean = $2.84\text{x}10^{-13}$ $m/cycle$ and *m* = 3.58.

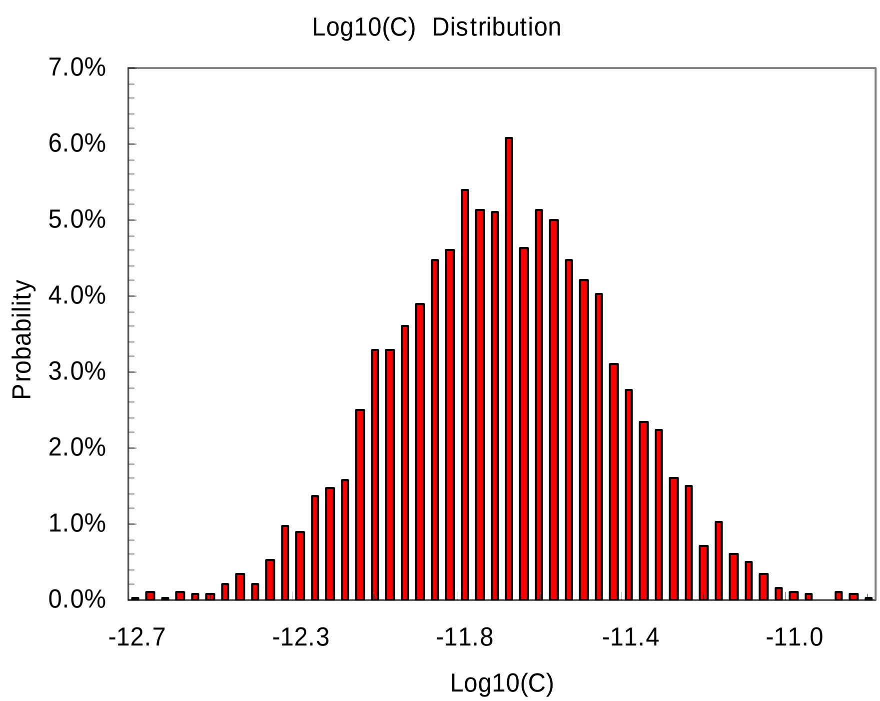

Fig. 4.5 Normal distribution of *Log(C)* for Incoloy 908
with *C* mean = $2.84\text{x}10^{-13}$ $m/cycle$, *m* = 3.58, cell number 50, cell size 0.04

Fracture toughness is one of the final fracture criteria, and is analyzed based on the published test data which are listed in Table 4.1. It follows student's t distribution due to its limited number of data. With total 36 specimens, there are 35 degree of freedom (DOF), mean value $=179.4\, MPa\sqrt{m}$, and standard deviation (std) $=45.3\, MPa\sqrt{m}$. 3000 random data of student's t distribution for the fracture toughness are drawn, and shown in Fig. 4.6.

Table 4.1 Measured data of fracture toughness K_{1c} for Incoloy 908

Specimen No.	$K_{1c}\,(MPa\sqrt{m})$	*Note*
1	265	Incoloy 908 handbook, Ref. 20
2	155.	
3	235.	
4	220.	
5	240.	
6	196.	
7	105.	
8	150.	
9	130.	
10	235	- welds
11	105	
12	150	
13	130	
14	214	
15	161	
16	266.	Nyilas database, Ref. 21
17	143.	
18	180.	
19	196.	
20	230.	
21	200.	
22	195.	
23	238.	
24	185.	
25	153.	
26	147.	
27	158.	
28	157.5	
29	200	- welds
30	166	
31	195	
32	114	
33	238	
34	131	
35	138	
36	138	

Mean: 179.4 $MPa\sqrt{m}$, std: 45.3 $MPa\sqrt{m}$

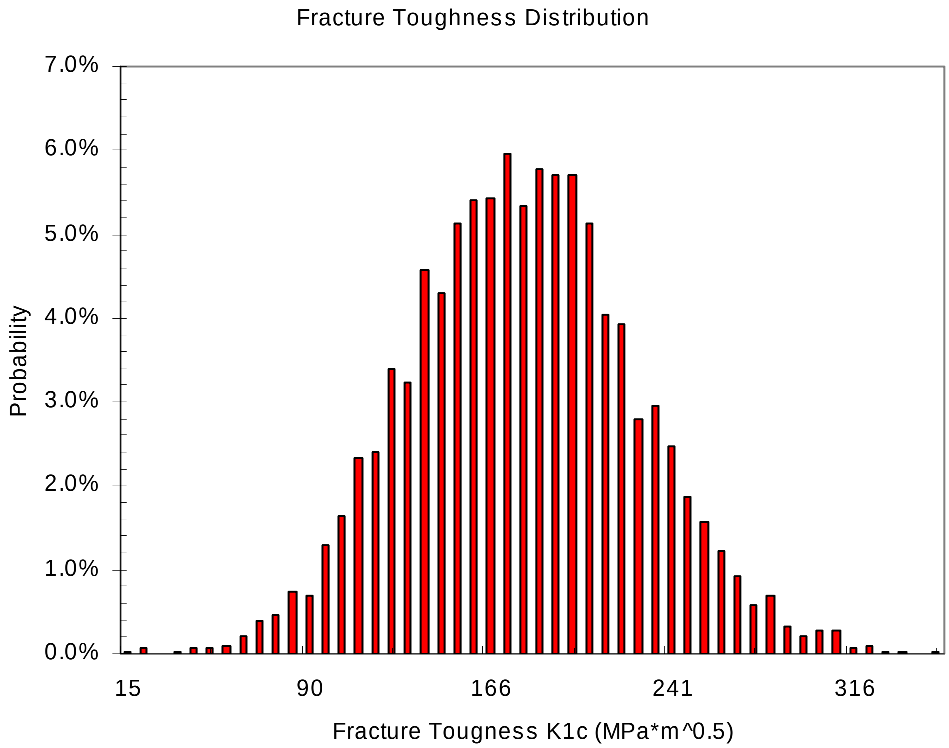

Fig. 4.6 Student's t distribution of fracture toughness for Incoloy 908, cell size 6.8 $MPa\sqrt{m}$

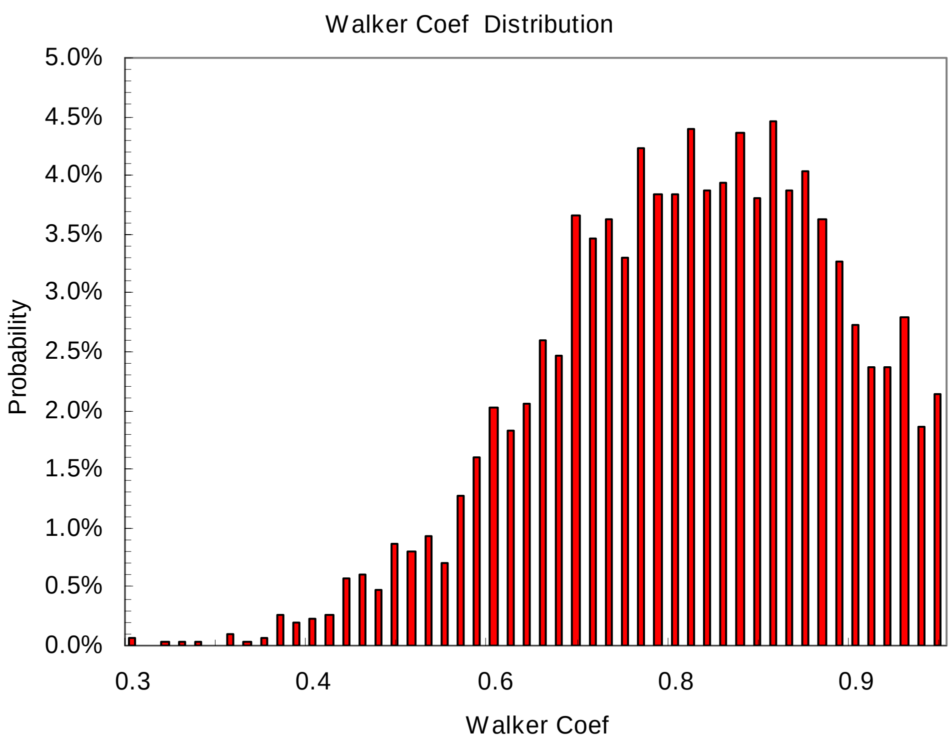

Fig. 4.7 Normal distribution of Walker coef. for load ratio effect, cell size 0.015

Walker coef. is a measure of the load ratio effect. It is estimated by analyzing the crack growth test data at R=0.1 and R=0.7 in Reference 7. By applying data regression and equation fitting we have mean value=0.79 and std=0.15. 3000 random data are drawn then for the normal distribution of Walker coef., and shown in Fig. 4.7.

4.1.1.3 Applied load

The applied stresses for CS conduit mad of Incoloy 908 are listed in Table 4.2. The total peak stresses are the sum of the operational and the residual. The operational stresses are obtained by several FEA analyses in parallel. The best match of these results between them approaches about 5%.[22] Therefore, a standard deviation of 5% is assumed for the calculated operational stresses in normal distribution. The residual stress comes from remaining stress due to conduit winding, its uncertainty is supposed to be larger than that of operational stress, hereby, we assume 0.1% for standard deviation for the residual stress. 3000 random normal data of the 1st, 2nd stress peaks and the residual are drawn, and shown in Figs. 4.8 to 4.10.

Table 4.2 Incoloy 908: Applied stresses for CS conduit

Process		Stress (MPa)	
		Min	Max
After winding and heat treatment		50	
During operation	1st peak	0	429
	2nd peak	0	401
Total peak stresses	1st peak	50	479
	2nd peak	50	451

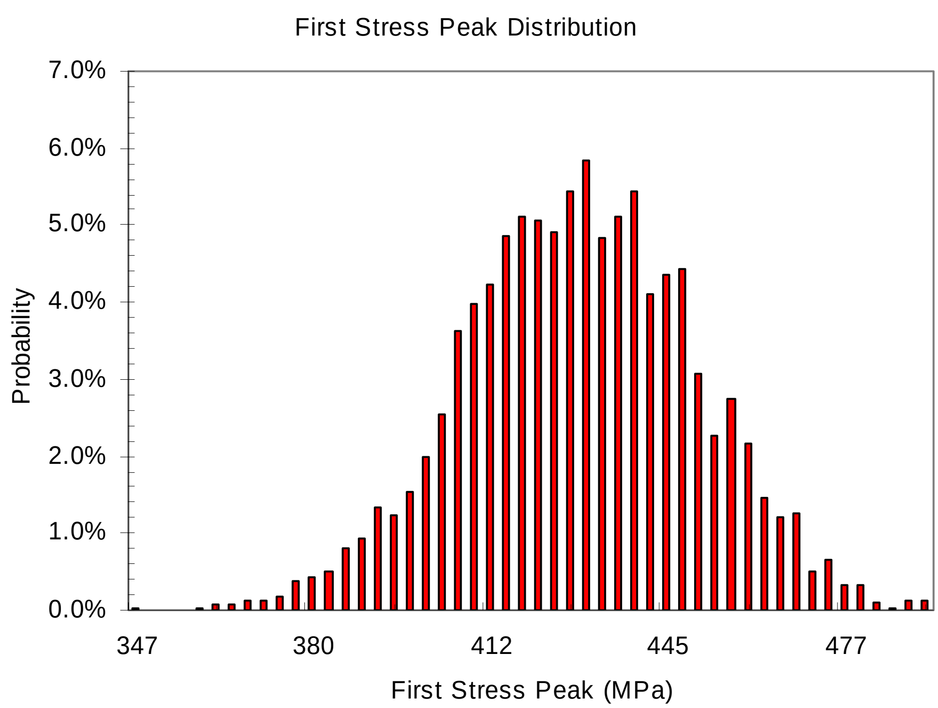

Fig. 4.8 Normal distribution of the first stress peak for Incoloy 908, cell size 2.97 MPa

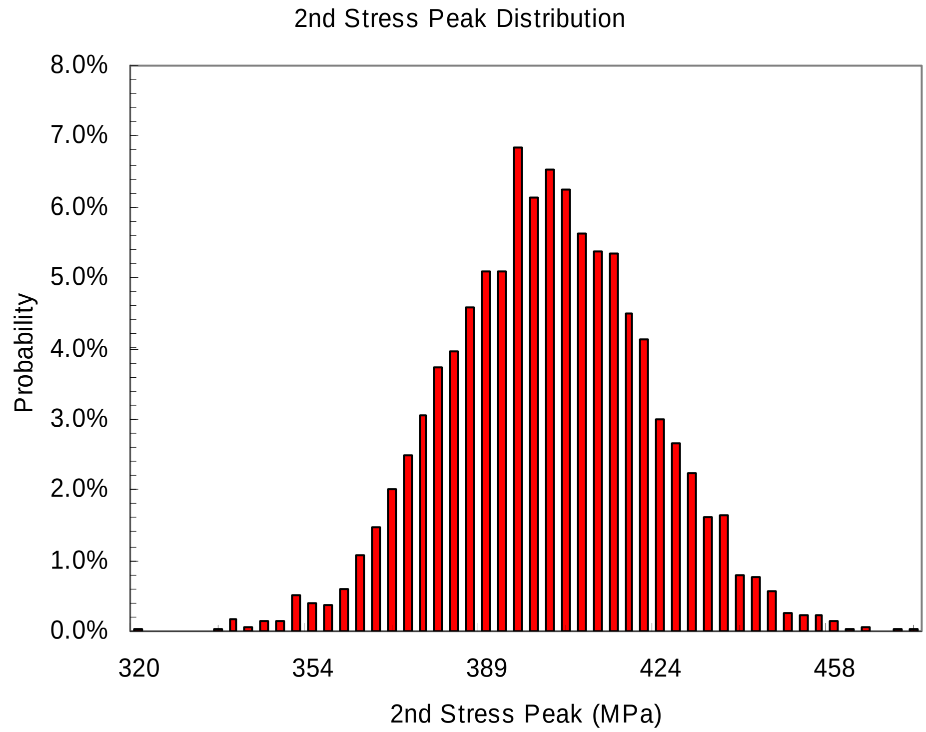

Fig. 4.9 Normal distribution of the 2nd stress peak for Incoloy 908, cell size 3.1 MPa

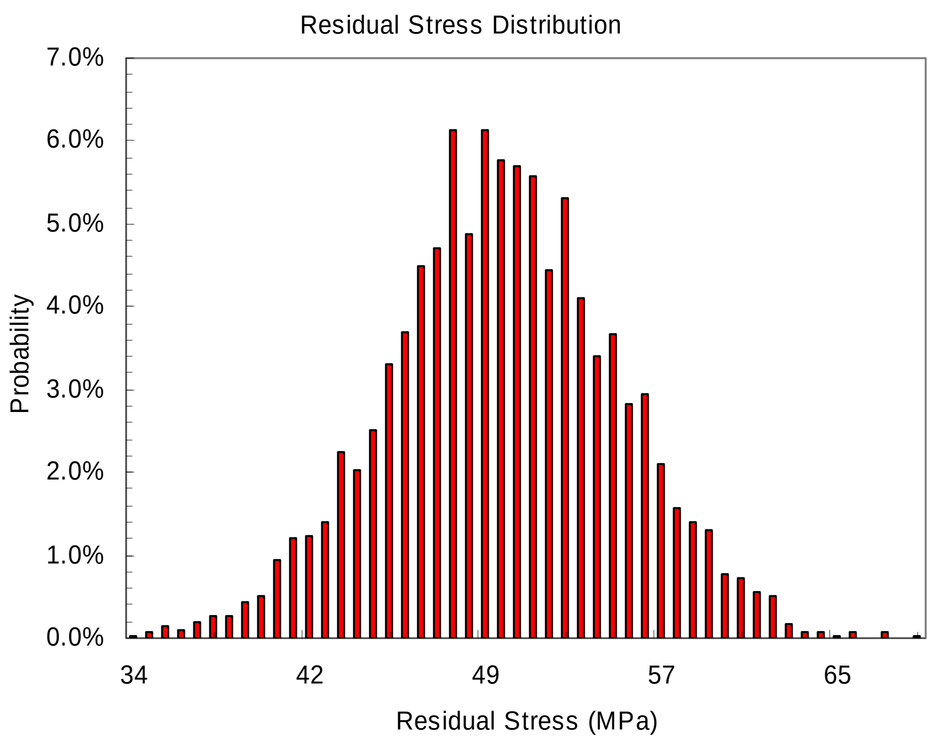

Fig. 4.10 Normal distribution of the residual stress for Incoloy 908, cell size 0.7 MPa

4.1.2 Output: fatigue life

3000 sets of random data were drawn from all independent variables as described above. Each set of data was inputted into a Fortran code to calculate a fatigue life by using Paris law integration. The results for a typical Paris parameter $C = 2.84\text{x}10^{-13}$ $m/cycle$, m =3.58 are shown in Fig. 4.11. Only the left part with small life is critical for this analysis. Therefore, the probability plot of *log(life)*, as shown in Fig. 4.12, is more useful than that of *life*. Both the probabilities of *life* and *log(life)* can be approximated by Weibull distribution.

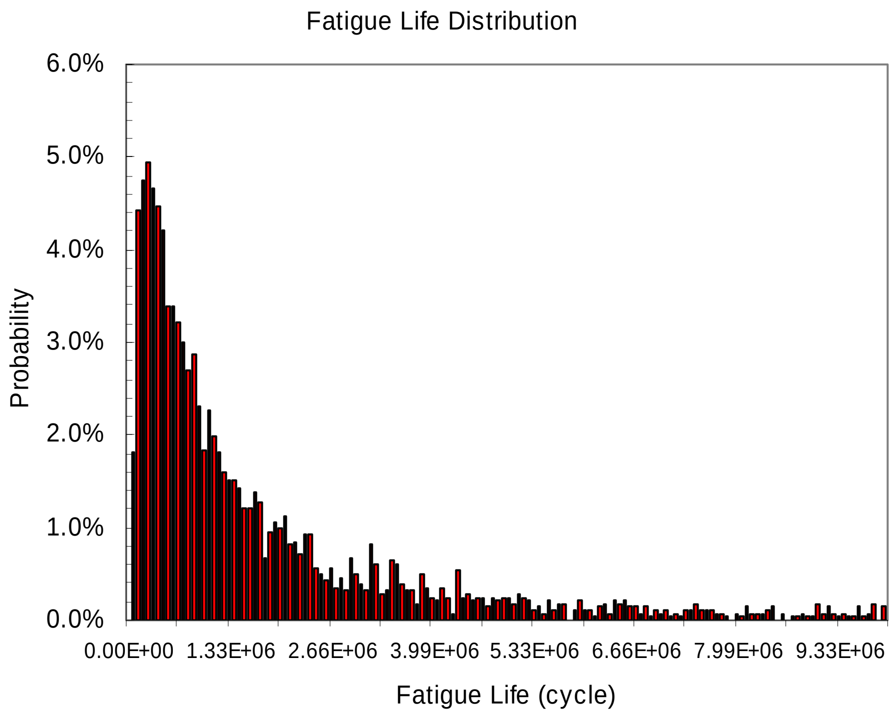

Fig. 4.11 Calculated distribution of fatigue *life* for Incoloy 908 with $C = 2.84\text{x}10^{-13}$ $m/cycle$, m =3.58, cell size 66679 cycles

The cumulative distribution $F(x)$ for x = *log(life)* is the integration of the density distribution $p(x)$, as shown in Fig. 4.12, over all *log(life)* span:

$$F(x) = \int_0^\infty p(x)dx \quad . \tag{4.4}$$

Fig. 4.13 shows $F(x)$ vs *log(life)*. It says that, for example, *log(life)* up to 5.5 (i.e., fatigue *life* from 0 to 316,228 cycles) has a probability of 18%.

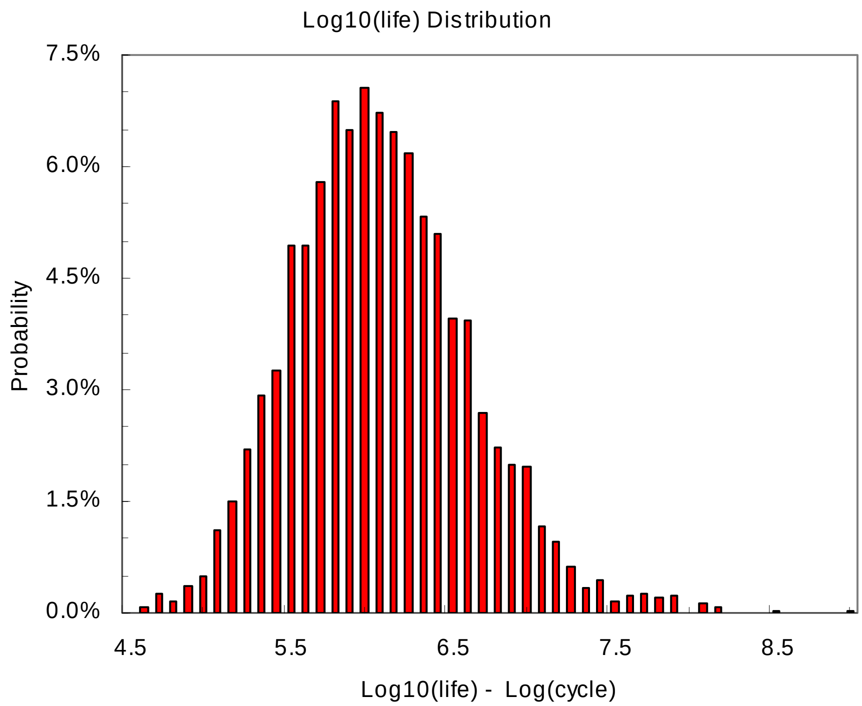

Fig. 4.12 Calculated distribution of *log(life)* for Incoloy 908 with C= 2.84x10^{-13} $m/cycle$, m=3.58, cell size 0.09

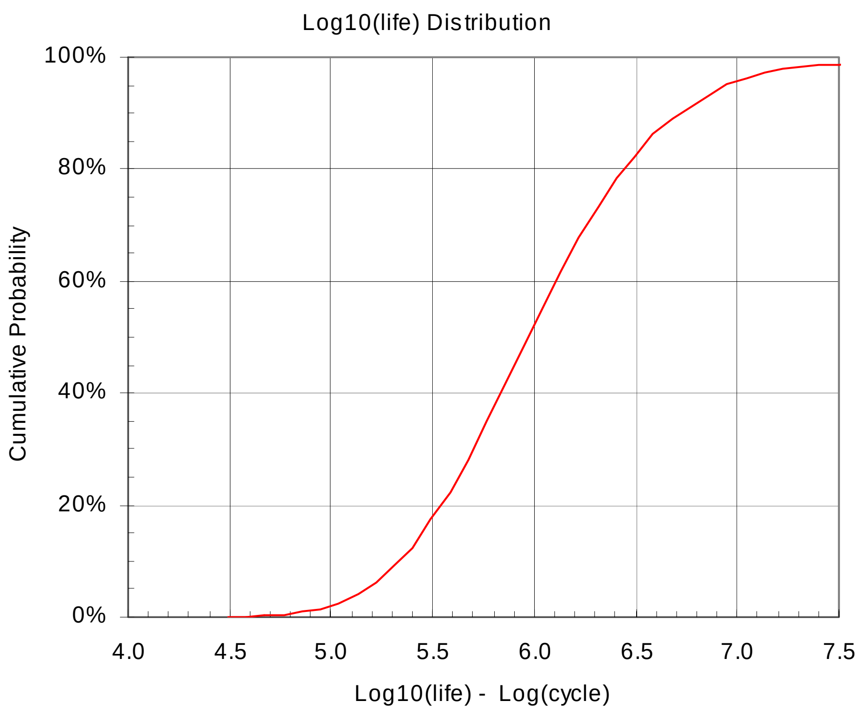

Fig. 4.13 Cumulative distribution function *F(x)* of *log(life)* for Incoloy 908 with C= 2.84x10^{-13} $m/cycle$, m=3.58

The reliability $R(x)$ for x=*log(life)* is:

$$R(x) = 1 - F(x), \qquad (4.5)$$

and is shown in Fig. 4.14. It says that, for example, the probability of survival over the fatigue *life* = 316,228 cycle or *log(life)* = 5.5 is 82%. The mean value of *log(life)* is located in 50%. The high end reliability located in the left side is particularly interesting for us, for example, reliabilities of 99.833%, 99.149%, 98.259%, which are more closed to the ITER design criteria.

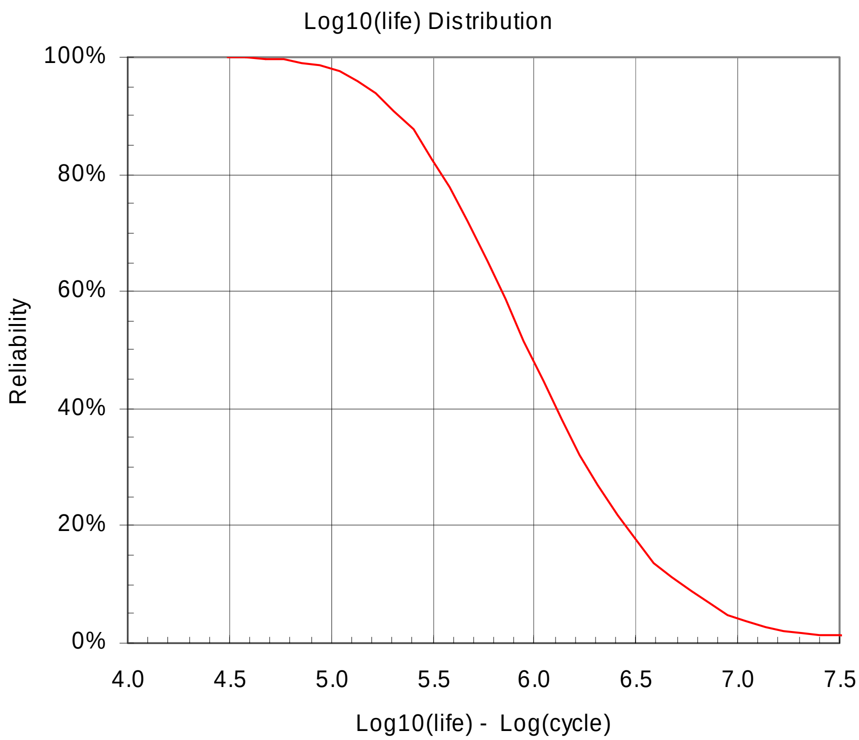

Fig. 4.14 Reliability distribution of *log(life)* for Incoloy 908
with C= 2.84x10^{-13} $m/cycle$, m=3.58

The probability results of either fatigue *life* or *log(life)* above are only for the typical Paris parameter C= 2.84x10^{-13} $m/cycle$ and m=3.58. So far, total 14 set of Paris parameters for Incoloy 908 have been collected and listed in Table 4.3. For each set of Paris parameter, a Monte Carlo simulation has been performed in the same way as described above. The results for each set of Paris parameter are shown in Fig. 4.15, and listed in Table 4.4. It is obvious that the results are different for different Paris parameters. The uncertainty associated with crack growth is estimated by the mean value over all results in Table 4.4, and some typical ones are listed in Table 4.5 for 3 selected reliabilities.

The No. 10 set in the database of total 15 Paris parameters for Incoloy 908 is: m=3.10, C=5.30x10^{-12} m/cycle (ref. to Table 4.3). It is found to be the results for short crack at 77K, and therefore not compactable with the database based on ASTM crack growth test standard for long crack at 4K. This set of data is therefore deleted from our statistical database, which leaves 14 set of Paris parameters and 13 degree of freedom (DOF) in Table 4.3.

Table 4.3 Incoloy 908: Paris parameters at 4K and R=0.1

Specimen N0.	*C x 10^{-12} (m/cycle)*	*m*	*Note*
1	1.97	3.03	Incoloy 908 Handbook, Ref. 20
2	0.39	3.68	
3	0.695	3.38	
4	2.13	3.03	
5	1.11	3.18	
6	1.56	3.22	
7	0.07	4.06	
8	1.99	3.04	
9	0.77	3.45	
11	0.04	4.68	
12	0.284	3.58	Jong's memo, Ref. 23
13	0.0546	3.92	
14	0.771	3.22	
15	0.131	3.72	

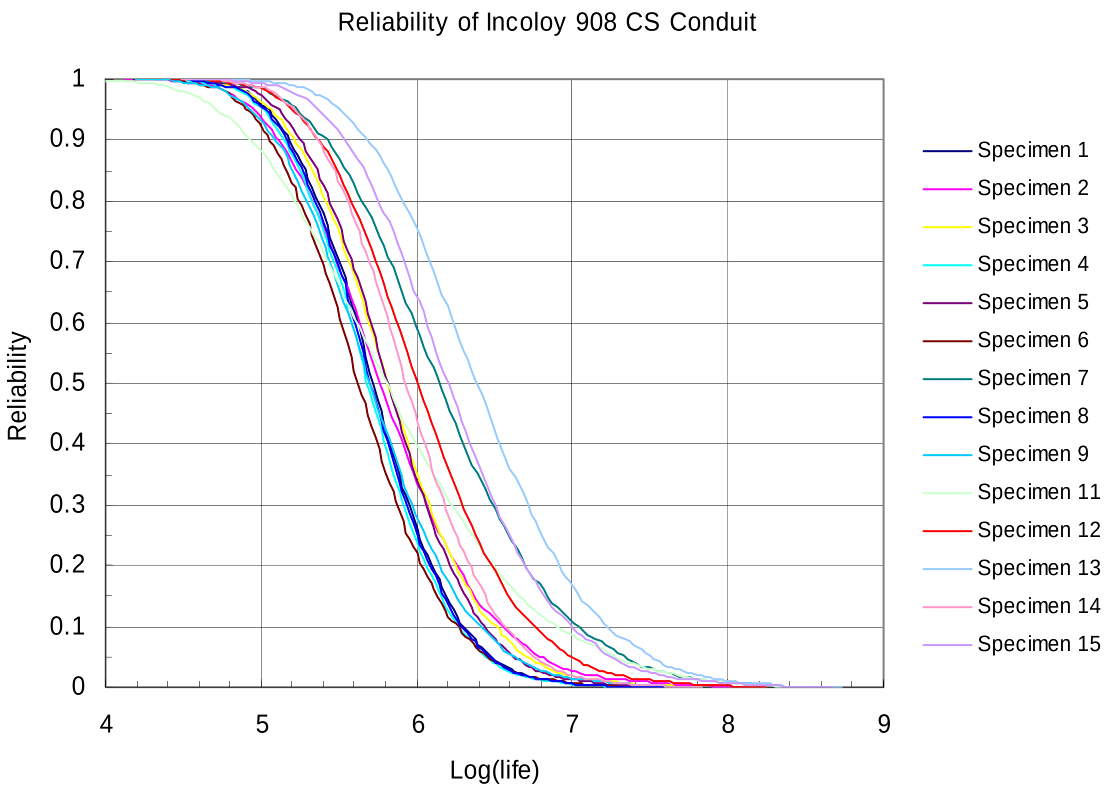

Fig. 4.15 Incoloy 908: Reliability vs. *log(life)* for each test specimen

Table 4.4 Incoloy 908: Results of *log(life)* based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	5.747373	0.4399155	4.584726	4.735909	4.88098
2	5.818972	0.5677671	4.401087	4.627683	4.756805
3	5.853202	0.503912	4.565996	4.751245	4.884806
4	5.713424	0.4398682	4.538278	4.702586	4.850109
5	5.845364	0.464695	4.637321	4.807853	4.938246
6	5.658103	0.4722685	4.432071	4.609041	4.742237
7	6.205476	0.638256	4.610396	4.88533	5.019578
8	5.732872	0.4416495	4.561512	4.722029	4.863836
9	5.741366	0.5183102	4.430662	4.608692	4.751124
11	5.885979	0.7684482	3.993684	4.328526	4.464126
12	6.050771	0.54595	4.678074	4.884332	5.019319
13	6.440131	0.604579	4.919786	5.168634	5.302254
14	5.964148	0.4723	4.740123	4.914178	5.046574
15	6.254634	0.57544	4.812868	5.039054	5.17309
Average	5.922273	0.532383	4.564756	4.770364	4.906649

Table 4.5 Incoloy 908: Uncertainty due to crack growth
based on the evaluation of mean over all crack growth data at 3 selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.357517
0.99149	1.151909
0.98259	1.015624

4.2 Uncertainty due to limited number of crack growth specimens

It is found that, from Table 4.4, the mean, standard deviation (std), and *log(life)* at given reliability are different for different Paris parameters from different test specimens. The uncertainty associated with the different Paris parameters from different specimens can be evaluated by small sample statistics using student's t distribution. According to Eq. 3.18, the uncertainty of one more observation for a data array with (n-1) degree of freedom at a given confidence limit or reliability is

$$u_s = t \cdot std \cdot \sqrt{1+\frac{1}{n}} \quad . \tag{4.6}$$

The degree of freedom for the current data set is 13 as listed in Table 4.4, mean=5.922273, and standard deviation (std)=0.23409 over the 13 means of *log(life)* . t values are obtained from student's t distribution for 3 selected reliabilities. The resulted uncertainties due to limited number of crack growth specimen are listed in Table 4.6.

Table 4.6 Incoloy 908: Uncertainty due to small sample statistics by student's t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794	0.23409	0.868134
0.99149	2.734955		0.662697
0.98259	2.356219		0.570927

Degree of freedom: 13

4.3 Minor effect due to other factors

The other minor factors, which are discussed in details in Reference 3, include short crack effect, crack closure effect, end-of-life factor, plastic zone on LEFM, T-stress effect, Z-stress effect, plate thickness, frequency, load shedding to insulation, bending from radial compliance of the insulation etc.. To obtain accurate value of uncertainty for each minor factor needs very extensive study for each factor. For sake of simplicity, we assume, based on previous data and lore, that the total uncertainty from all other minor factors is 10% of those for the crack growth, i.e., standard deviation ≈ 0.05. The scale factors relative to the standard deviation for different reliabilities are estimated based on normal distribution. The results are listed in Table 4.7.

Table 4.7 Incoloy 908: Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~ 0.05

4.4 Reliability and volume effect

The total uncertainty due to crack growth, small sampling, and minor effect is the square root of sum of square over all individual uncertainties:

$$u_{tot} = \sqrt{u_c^2 + u_s^2 + u_m^2} \tag{4.7}$$

The *log(life)* for a given reliability is then obtained by mean minus the total uncertainty:

$$\log(life) = \overline{\log(life)} - u_{tot} \ . \tag{4.8}$$

The results are listed in Table 4.8 for 3 selected reliabilities.

Table 4.8 Incoloy 908: Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	5.9222725	1.357517	0.868134	0.15	1.618335	4.303937	20134.34
0.99149		1.151909	0.662697	0.12	1.334339	4.587933	38719.79
0.98259		1.015624	0.570927	0.11	1.170277	4.751995	56493.05

However, the above analysis is assumed to be for one CS coil only. As to the whole CS system with 6 coils, the reliability should be:

$$R(CS) = R(coil)^6 \ . \tag{4.9}$$

The results of reliability for whole CS system are listed in Table 4.9.

Table 4.9 Incoloy 908: Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.303937	20134
0.99149	0.95	4.587933	38720
0.98259	0.90	4.751995	56493

4.5 Effect of applied stress on reliability

The effect of applied stresses on the reliability of ITER CS conduit made of Incoloy 908 has been studied. 5 cases with different reductions of operation stresses are simulated, and summarized in Table 4.10. A crack area of $0.75 mm^2$ for 95% probability is assumed for all cases. The cut-off life during simulation is 10^9 stress cycles.

Table 4.10 Incoloy 908: Simulation cases for stress effect on fatigue life

Case No.	Percentage of nominal operation stress	Residual stress (MPa)	Peak operation stresses (MPa)	Total peak stresses (MPa)
1	100%	50	429 / 401	479 / 451
2	90%	50	386.1 / 360.9	436.1 / 410.9
3	80%	50	343.2 / 320.8	303.2 / 370.8
4	75%	50	321.75 / 300.75	371.75 / 350.75
5	70%	50	300.3 / 280.7	350.3 / 330.7

Note: Crack area of $0.75 mm^2$ for 95% probability, cut-off *life*=10^9 cycles

The detailed simulation results are listed in Appendix 1. A summary is listed in Table 4.11, and shown in Figure 4.16, in which fatigue life is a function of the stress reduction at 3 selected reliabilities. At 90% reliability, it gives 60,000 cycles for 98.5% nominal operation stress, or 120,000 cycles for 78.8% nominal operation stress

Table 4.11 Incoloy 908: Fatigue life of CS conduit
at 3 selected reliabilities for different reductions of stress

Simulation case No.	*Percentage of nominal operation stress*	*Reliability for 6 coils $R(CS)$*		
		99%	95%	90%
1	100%	20134	38720	56493
2	90%	27616	53742	78964
3	80%	39029	76895	113651
4	75%	46995	93832	139110
5	70%	57488	116255	171915

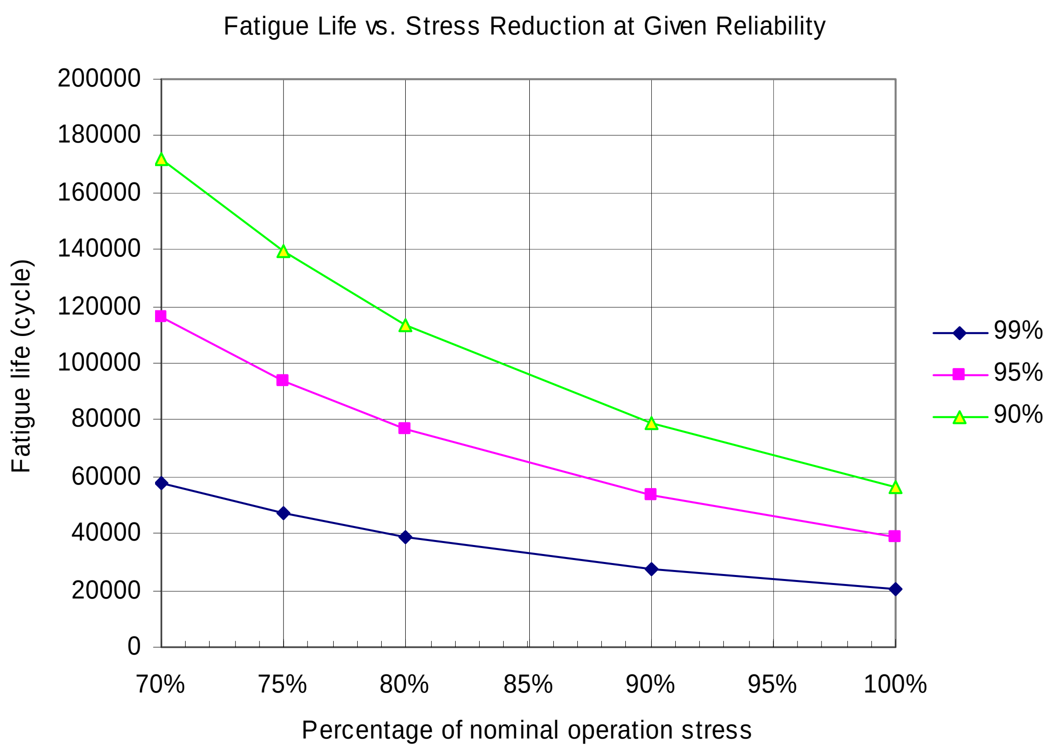

Fig. 4.16 Incoloy 908: Fatigue life vs. percentage of nominal operation stress
for 3 selected reliabilities

5. PROBABILISTIC ANALYSIS ON CS CONDUIT MADE OF JK2LB

The Probabilistic analysis on CS conduit made of JK2LB has been performed based on the same principles and methods used for Incoloy 908. There are 3 major differences of input variables for JK2LB against Incoloy 908: (a) Paris parameters and the number of specimen; (b) The fracture toughness and the number of specimen; (c) Applied stresses. For the rest of input variables without available test data, we assume that their distributions are the same with Incoloy 908.

5.1 Major effect associated with fatigue crack growth

5.1.1 Input variables

The distributions of crack configurations including size, shape and location are assumed to be the same with those of Incoloy 908.

The available Paris parameters of JK2LB are listed in Table 5.1, and the fracture toughness in Table 5.2. The distribution of Walker coef is assumed to be the same with Incoloy 908.

The applied stresses are different from Incoloy 908, and listed in Table 5.3.

Table 5.1 JK2LB: Paris parameters for fatigue crack growth at 4K[25]

Specimen No	Condition	C (m/cycle)	m
1	Aged (TL)	3.053e-15	4.828
2	Aged (TL)	5.253e-15	4.692
3	Aged (LT)	1.635e-14	4.317
4	Weld	5.284e-14	4.034
5	Weld	3.536e-13	3.515

Note: Aged at 650C x 240h

Table 5.2 JK2LB: Fracture toughness at 4K[25]

Specimen No	Condition	$K_{1c}(MPa\sqrt{m})$
1	HE&CD Aged (TL)	91
2	HE&CD Aged (TL)	92
3	HE&CD Aged (LT)	95
4	HE&CD Aged (LT)	103
5	Weld Aged	157
6	Weld Aged	163

Note: Aged at 650C x 240h, HE: hot extruded, CD: cold drawn

Table 5.3 JK2LB: Applied stresses for CS conduit

Process		Stress (MPa)	
		Min	Max
After winding and heat treatment		25	
During operation	1^{st} peak	0	470
	2^{nd} peak	0	440
Total peak stresses	1^{st} peak	25	495
	2^{nd} peak	25	465

5.1.2 Output: fatigue life

3000 sets of random data were drawn from all independent variables. Each set of data was inputted into a Fortran code to calculate a fatigue life by using Paris law integration. The simulation results and the reliability against *log(life)* for each specimen are shown in Fig. 5.1, and listed in Table 5.4. The uncertainties due to crack growth at 3 selected reliabilities are listed in Table 5.5 based on the evaluation of mean over all crack growth data in Table 5.4.

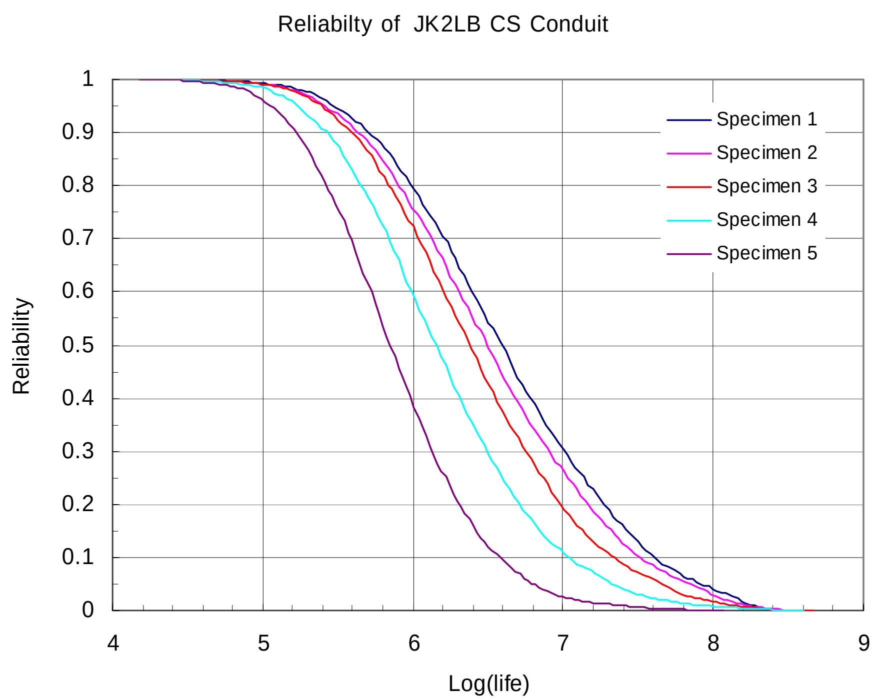

Fig. 5.1 JK2LB: Reliability vs. *log(life)* for each test specimen

Table 5.4 JK2LB: Results of *log(life)* at 3 selected reliabilities based on fatigue crack growth for each test specimen

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	6.641738	0.727904	4.745484	5.04562	5.221428
2	6.553269	0.721269	4.678945	4.975313	5.162572
3	6.439263	0.675403	4.69731	4.983243	5.166094
4	6.212121	0.637051	4.582873	4.854553	5.02669
5	5.893075	0.536546	4.473236	4.697588	4.870862
Average	6.347893	0.659634	4.63557	4.911263	5.089529

Table 5.5 JK2LB: Uncertainty due to fatigue crack growth based on the evaluation of mean over all crack growth data at 3 selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.712324
0.99149	1.43663
0.98259	1.258364

5.2 Uncertainty due to limited number of crack growth specimen

The uncertainty due to limited number of crack growth specimen is evaluated by the small sampling statistics described in Section 3.2. The results are listed in Table 5.6. The critical values "t" are much larger than those for Incoloy 908 due to less number of test specimen for JK2LB.

Table 5.6 JK2LB: Uncertainty due to small sample statistics

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty* u_s
0.99833	6.250682	0.300901193	2.060354438
0.99149	3.93579		1.297318019
0.98259	3.140848		1.035288648

Degree of freedom: 4

5.3 Minor effect due to other factors

The evaluation of minor effect due to other factors is almost identical to Incoloy 908 except that the standard deviation slightly increases to ~0.07, 10% of the mean standard deviation of fatigue life as listed in Table 5.4. The results of uncertainty for minor effect are listed in Table 5.7.

Table 5.7 JK2LB: Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17

0.98259	2.11	0.15

Std ~ 0.07

5.4 Reliability and volume effect

The total uncertainty due to all factors including crack growth, small sample statistics and minor effect is obtained by Eq. 4.7. The results of *log(life)* at 3 selected reliabilities are obtained by Eq. 4.8. All the above values are listed in Table 5.8. The volume effect is evaluated by Eq. 4.9, and the final results of fatigue life are listed in Table 5.9.

Table 5.8 JK2LB: Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.3478932	1.712324	2.060354	0.21	2.687231	3.660662	4577.853
0.99149		1.43663	1.297318	0.17	1.943152	4.404741	25394.6
0.98259		1.258364	1.035289	0.15	1.636399	4.711494	51462.86

Table 5.9 JK2LB: Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	3.660662	4578
0.99149	0.95	4.404741	25395
0.98259	0.90	4.711494	51463

5.5 Effect of applied stress on reliability

The effect of applied stresses on the reliability of ITER CS conduit made of JK2LB has been studied. 5 cases with different reductions of operation stresses are simulated, and summarized in Table 5.10. A crack area of 0.75mm^2 for 95% probability is assumed for all cases. The cut-off *life* during simulation is 10^9 stress cycles.

Table 5.10 JK2LB: Simulation cases for stress effect on fatigue life

Case No.	Percentage of nominal operation stress	Residual stress (MPa)	Peak operation stresses (MPa)	Total peak stresses (MPa)
1	100%	25	470 / 440	495 / 465
2	90%	25	423 / 396	448 / 421
3	80%	25	376 / 352	401 / 377
4	75%	25	352.5 / 330	377.5 / 355
5	70%	25	329 / 308	354 / 333

Note: Crack area of 0.75 mm^2 for 95% probability, cut-off *life*=10^9 cycles

The detailed simulation results are listed in Appendix 2. A summary is listed in Table 5.11, and shown in Figure 5.2, in which fatigue life is a function of the stress reduction at 3 selected reliabilities. At 90% reliability, it gives 60,000 cycles for 97% nominal operation stress, or 120,000 cycles for 80% nominal operation stress

Table 5.11 JK2LB: Fatigue life of CS conduit
at 3 selected reliabilities for different reductions of stress

Simulation case No.	*Percentage of nominal operation stress*	*Reliability for 6 coils $R(CS)$*		
		99%	95%	90%
1	100%	4578	25395	51463
2	90%	6237	36631	75325
3	80%	9153	57040	117469
4	75%	11270	72272	148880
5	70%	14366	92754	192631

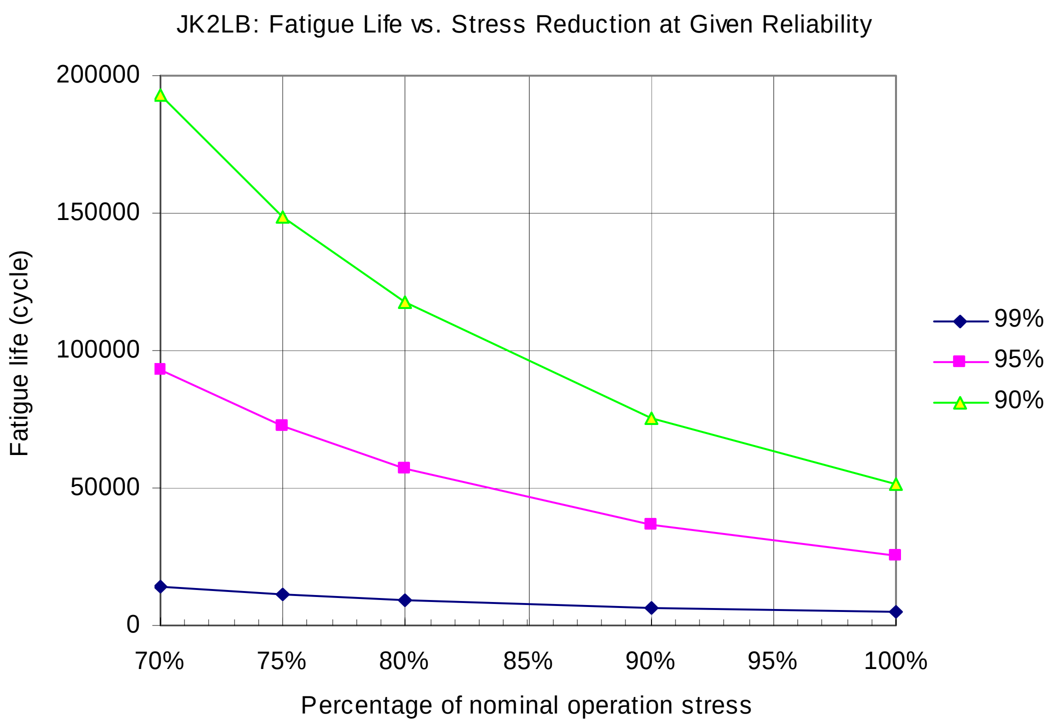

Fig. 5.2 JK2LB: Fatigue life vs. percentage of nominal operation stress
for 3 selected reliabilities

6. DISCUSSION AND CONCLUSIONS

a. A probabilistic study has been performed for ITER conduit made of either Incoloy 908 or JK2LB. The results of stress fatigue cycle at given reliability for the nominal design stresses are summarized in Table 6.1.

Table 6.1 Stress fatigue cycle at given reliability

Reliability	Incoloy 908	JK2LB
99%	20134	4578
95%	38720	25395
90%	56493	51463

The above results are obtained based on existing test data, reasonable assumptions and a best guess of some variables without available data. If the best guess or any assumptions are changed, the results will change.

The stress raisers in the joint region are not included in this analysis. It may will reduce the fatigue life for given reliabilities, but it is not yet know whether this reduction will be significant. Therefore, a further reduction of applied stress may be necessary in order to get required reliability.

b. A comparison of the fatigue life under 5 different reductions of nominal stresses between JK2LB and Incoloy 908 is shown in Fig. 6.1. It is found that both materials have similar fatigue life at 90% reliability, but Incoloy 908 shows much better fatigue behavior at higher reliabilities. Part of the reason comes from the lower fracture toughness K_{1c} of JK2LB, and its limited K_{1c} database gives JK2LB much higher life uncertainty. The limited database for the Paris parameters of JK2LB greatly increases the uncertainty of life due to small sample statistics. It indicates a need to test more JK2LB in order to improve statistical estimates of its fatigue life.

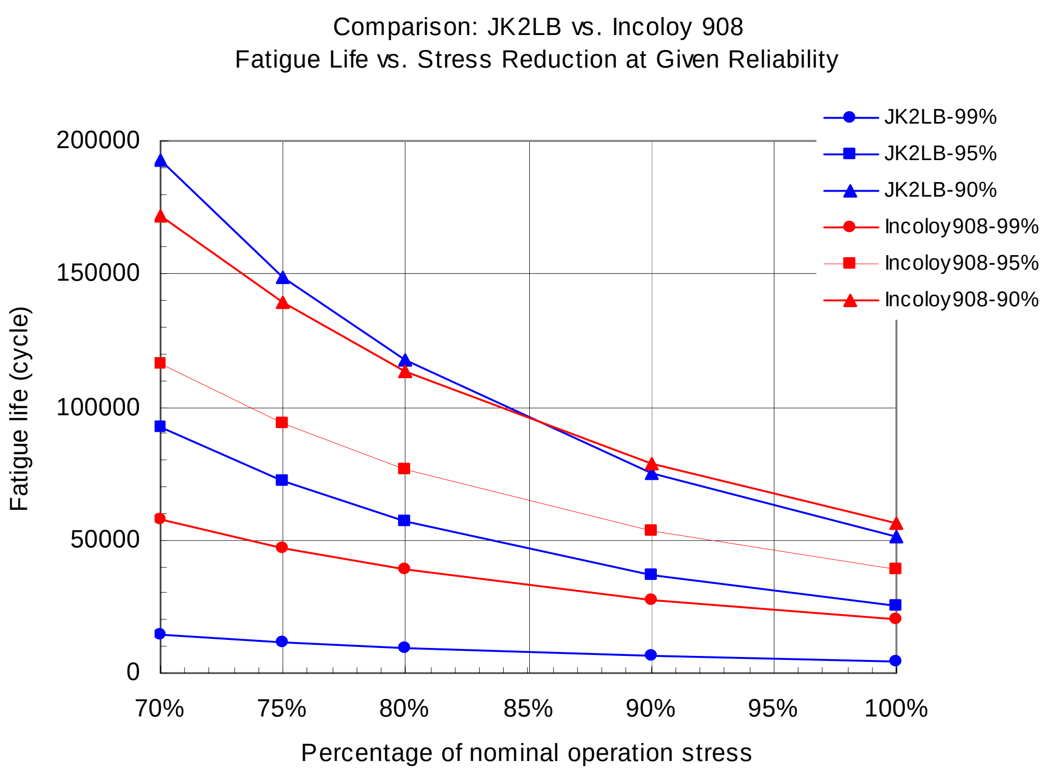

Fig. 6.1 Fatigue life vs. percentage of nominal operation stress at 3 selected reliabilities for both Incoloy 908 and JK2LB

c. A recent report by Titus[22] for the structural analysis of the CS conduit gives much lower hoop stress without the residual stress: ~350MPa for JK2LB and ~334 MPa for Incoloy 908 at ITER precharge condition. These new hoop stress data are about 25% lower that those listed in the previous design documents.[1,23] However, the statistical results for the reduced stresses are still included in Fig. 6.1.

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APPENDIX 1 INCOLOY 908: EFFECT OF APPLIED STRESS ON RELIABILITY

1. Introduction

The effect of stresses on the reliability of Incoloy 908 CS conduit is analyzed. The results are reported hereafter. All tables are labeled with reference to those in Sec. 4. Five cases with different operational stresses are simulated. A crack area of 0.75mm^2 for 95% probability is assumed for all cases. The cut-off life during simulation is 10^9 stress cycles.

2. Results for 100% Nominal Operation Stresses

Table 4.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	5.747373	0.4399155	4.584726	4.735909	4.88098
2	5.818972	0.5677671	4.401087	4.627683	4.756805
3	5.853202	0.503912	4.565996	4.751245	4.884806
4	5.713424	0.4398682	4.538278	4.702586	4.850109
5	5.845364	0.464695	4.637321	4.807853	4.938246
6	5.658103	0.4722685	4.432071	4.609041	4.742237
7	6.205476	0.638256	4.610396	4.88533	5.019578
8	5.732872	0.4416495	4.561512	4.722029	4.863836
9	5.741366	0.5183102	4.430662	4.608692	4.751124
11	5.885979	0.7684482	3.993684	4.328526	4.464126
12	6.050771	0.54595	4.678074	4.884332	5.019319
13	6.440131	0.604579	4.919786	5.168634	5.302254
14	5.964148	0.4723	4.740123	4.914178	5.046574
15	6.254634	0.57544	4.812868	5.039054	5.17309
Average	5.922273	0.532383	4.564756	4.770364	4.906649

Table 4.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.357517
0.99149	1.151909
0.98259	1.015624

Table 4.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794		0.868134

0.99149	2.734955	0.23409	0.662697
0.98259	2.356219		0.570927

Degree of freedom: 13

Table 4.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~ 0.05

Table 4.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	5.9222725	1.357517	0.868134	0.15	1.618335	4.303937	20134.34
0.99149		1.151909	0.662697	0.12	1.334339	4.587933	38719.79
0.98259		1.015624	0.570927	0.11	1.170277	4.751995	56493.05

Table 4.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.303937	20134
0.99149	0.95	4.587933	38720
0.98259	0.90	4.751995	56493

3. Results for 90% Nominal Operation Stresses

Table 4.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	5.882463	0.439851	4.727989	4.872761	5.017388
2	5.983005	0.567817	4.55955	4.784901	4.917696
3	6.00387	0.503887	4.714746	4.903225	5.038262
4	5.848522	0.439805	4.681628	4.841869	4.981615
5	5.987782	0.465912	4.77831	4.945469	5.079633
6	5.801646	0.472229	4.582373	4.755723	4.890963
7	6.381948	0.631073	4.796195	5.060473	5.195638
8	5.868414	0.441587	4.704789	4.858251	4.997552
9	5.895141	0.518289	4.5752	4.773333	4.912214
11	6.088078	0.75945	4.202374	4.529931	4.680901
12	6.210346	0.545964	4.838066	5.037417	5.182474
13	6.608173	0.594494	5.091944	5.343262	5.476836

14	6.107702	0.472253	4.882667	5.059512	5.191089
15	6.417527	0.57026	4.980943	5.215689	5.341478
Average	6.077473	0.530205	4.722627	4.927273	5.064553

Table 4.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.354846
0.99149	1.1502
0.98259	1.01292

Table 4.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794	0.244089	0.905213
0.99149	2.734955		0.691002
0.98259	2.356219		0.595312

Degree of freedom: 13

Table 4.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~ 0.05

Table 4.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.077472643	1.354846	0.905213	0.15	1.636313	4.44116	27615.95
0.99149		1.1502	0.691002	0.12	1.347161	4.730311	53741.67
0.98259		1.01292	0.595312	0.11	1.180044	4.897429	78963.95

Table 4.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil R(coil)	*Reliability for 6 coils R(CS)*	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.44116	27616
0.99149	0.95	4.730311	53742
0.98259	0.90	4.897429	78964

4. Results for 80% Nominal Operation Stresses

Table 4.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R*= 0.99833	*R*=0.99149	*R*=0.98259
1	6.033022	0.439861	4.861813	5.027137	5.164168
2	6.165826	0.56791	4.744759	4.963305	5.100343
3	6.171804	0.503925	4.889918	5.068683	5.207462
4	5.999083	0.439816	4.824971	4.990149	5.134315
5	6.145131	0.46467	4.937296	5.108175	5.240948
6	5.961622	0.472263	4.740127	4.913757	5.04886
7	6.576361	0.62077	5.002034	5.266908	5.400561
8	6.019462	0.441598	4.861722	5.009086	5.153905
9	6.066531	0.518338	4.755384	4.941477	5.077052
11	6.307892	0.743039	4.448678	4.76859	4.902632
12	6.387513	0.544773	5.018234	5.218648	5.356148
13	6.79689	0.586273	5.288877	5.538144	5.67016
14	6.267689	0.472276	5.041214	5.217649	5.350984
15	6.59908	0.565169	5.173727	5.385406	5.527991
Average	6.24985	0.527192	4.899197	5.101222	5.238252

Table 4.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.350654
0.99149	1.148628
0.98259	1.011598

Table 4.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794	0.256341	0.950651
0.99149	2.734955		0.725687
0.98259	2.356219		0.625194

degree of freedom =13

Table 4.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~0.05

Table 4.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.249850429	1.350654	0.950651	0.15	1.658464	4.591386	39028.91
0.99149		1.148628	0.725687	0.12	1.363953	4.885897	76894.85
0.98259		1.011598	0.625194	0.11	1.194278	5.055573	113650.9

Table 4.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.591386	39029
0.99149	0.95	4.885897	76895
0.98259	0.90	5.055573	113651

5. Results for 75% Nominal Operation Stresses

Table 4.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	$R=$ 0.99833	$R=$0.99149	$R=$0.98259
1	6.115303	0.439903	4.945101	5.110252	5.247435
2	6.265015	0.566656	4.849466	5.057335	5.198647
3	6.263586	0.503983	4.972056	5.157181	5.302912
4	6.081362	0.439856	4.921819	5.088266	5.215138
5	6.23149	0.464712	5.024923	5.194596	5.329366
6	6.049063	0.472305	4.821792	4.99989	5.133712
7	6.683431	0.616484	5.113523	5.379925	5.511269
8	6.102023	0.441641	4.934293	5.087412	5.236973
9	6.160215	0.518399	4.846869	5.041625	5.17546
11	6.425926	0.732141	4.570165	4.884943	5.035269
12	6.484721	0.544832	5.113196	5.32479	5.45357
13	6.897798	0.57952	5.400953	5.643784	5.783341
14	6.355123	0.472328	5.12808	5.303334	5.438692
15	6.695661	0.558461	5.273991	5.499676	5.626523
Average	6.343623	0.525087	4.994016	5.198072	5.334879

Table 4.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.349606
0.99149	1.145551
0.98259	1.008744

Table 4.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794	0.26285	0.97479
0.99149	2.734955		0.744114
0.98259	2.356219		0.641069

degree of freedom =13

Table 4.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~0.05

Table 4.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.343623	1.349606	0.97479	0.15	1.671572	4.672051	46994.91
0.99149		1.145551	0.744114	0.12	1.371274	4.972349	93831.6
0.98259		1.008744	0.641069	0.11	1.200264	5.143359	139110.2

Table 4.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.672051	46995
0.99149	0.95	4.972349	93832
0.98259	0.90	5.143359	139110

6. Results for 70% Nominal Operation Stresses

Table 4.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	6.2031	0.439953	5.041939	5.208135	5.334591
2	6.371626	0.566728	4.954998	5.183822	5.30629
3	6.3615	0.504042	5.064127	5.264619	5.398352
4	6.169149	0.439911	5.005177	5.170818	5.300466
5	6.323627	0.464774	5.117433	5.283947	5.418896
6	6.14235	0.472368	4.919485	5.08972	5.226887
7	6.795188	0.609278	5.228616	5.495955	5.631159
8	6.190086	0.441691	5.022795	5.179091	5.320919
9	6.260157	0.518459	4.942378	5.138659	5.275432
11	6.549421	0.718988	4.710496	5.023486	5.163752
12	6.586379	0.541287	5.215603	5.428002	5.558927
13	7.001509	0.568187	5.511556	5.762944	5.888251
14	6.448411	0.472375	5.220641	5.398893	5.53393
15	6.799213	0.552272	5.381697	5.59196	5.734038
Average	6.44298	0.522165	5.095496	5.301432	5.435135

Table 4.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.347484
0.99149	1.141548
0.98259	1.007845

Table 4.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of Mean log(life)*	*Uncertainty* u_s
0.99833	3.582794	0.269057	0.997808
0.99149	2.734955		0.761685
0.98259	2.356219		0.656207

degree of freedom =13

Table 4.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.15
0.99149	2.39	0.12
0.98259	2.11	0.11

Std ~0.05

Table 4.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.442979714	1.347484	0.997808	0.15	1.6834	4.75958	57488.38
0.99149		1.141548	0.761685	0.12	1.377568	5.065411	116254.9
0.98259		1.007845	0.656207	0.11	1.207667	5.235313	171914.7

Table 4.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.75958	57488
0.99149	0.95	5.065411	116255
0.98259	0.90	5.235313	171915

APPENDIX 1 JK2LB: EFFECT OF APPLIED STRESS ON RELIABILITY

1. Introduction

The effect of stresses on the reliability of JK2LB CS conduit is analyzed. The results are reported hereafter. All tables are labeled with reference to those in Sec. 5. Five cases with different operational stresses are simulated. A crack area of 0.75mm^2 for 95% probability is assumed for all cases. The cut-off life during simulation is 10^9 stress cycles.

2. Results for 100% Nominal Operation Stresses

Table 5.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	6.641738	0.727904	4.745484	5.04562	5.221428
2	6.553269	0.721269	4.678945	4.975313	5.162572
3	6.439263	0.675403	4.69731	4.983243	5.166094
4	6.212121	0.637051	4.582873	4.854553	5.02669
5	5.893075	0.536546	4.473236	4.697588	4.870862
Average	6.347893	0.659634	4.63557	4.911263	5.089529

Table 5.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty u_c*
0.99833	1.712324
0.99149	1.43663
0.98259	1.258364

Table 5.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty u_s*
0.99833	6.250682	0.300901193	2.060354438
0.99149	3.93579		1.297318019
0.98259	3.140848		1.035288648

Degree of freedom: 4

Table 5.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17
0.98259	2.11	0.15

Std ~ 0.07

Table 5.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.3478932	1.712324	2.060354	0.21	2.687231	3.660662	4577.853
0.99149		1.43663	1.297318	0.17	1.943152	4.404741	25394.6
0.98259		1.258364	1.035289	0.15	1.636399	4.711494	51462.86

Table 5.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil R(coil)	*Reliability for 6 coils R(CS)*	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	3.660662	4578
0.99149	0.95	4.404741	25395
0.98259	0.90	4.711494	51463

3. Results for 90% Nominal Operation Stresses

Table 5.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	6.823185	0.694708	4.929363	5.245073	5.43611
2	6.742806	0.698779	4.89358	5.18665	5.371662
3	6.622606	0.661399	4.878987	5.174698	5.356171
4	6.389051	0.628814	4.77486	5.043652	5.208452
5	6.052618	0.535353	4.678899	4.872861	5.03239
Average	6.526053	0.64381	4.831138	5.104587	5.280957

Table 5.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.694915
0.99149	1.421466
0.98259	1.245096

Table 5.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty* u_s
0.99833	6.250682	0.311248	2.131204
0.99149	3.93579		1.341929
0.98259	3.140848		1.070889

Degree of freedom: 4

Table 5.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17
0.98259	2.11	0.15

Std ~ 0.07

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Table 5.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.5260532	1.694915	2.131204	0.21	2.731093	3.79496	6236.778
0.99149		1.421466	1.341929	0.17	1.962203	4.56385	36631.13
0.98259		1.245096	1.070889	0.15	1.649111	4.876942	75325.47

Table 5.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil R(coil)	*Reliability for 6 coils R(CS)*	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	3.79496	6237
0.99149	0.95	4.56385	36631
0.98259	0.90	4.876942	75325

4. Results for 80% Nominal Operation Stresses

Table 5.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	7.024678	0.65667	5.202069	5.508309	5.683776
2	6.942479	0.662791	5.130858	5.429691	5.616438
3	6.824147	0.641004	5.118675	5.416441	5.577369
4	6.586302	0.617076	4.990064	5.250927	5.417376
5	6.230482	0.534619	4.859286	5.064323	5.209337
Average	6.721618	0.622432	5.06019	5.333938	5.500859

Table 5.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.661427
0.99149	1.387679
0.98259	1.220758

Table 5.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty* u_s
0.99833	6.250682	0.320415	2.193968
0.99149	3.93579		1.381449
0.98259	3.140848		1.102427

degree of freedom =4

Table 5.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17
0.98259	2.11	0.15

Std ~0.07

Table 5.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.7216176	1.661427	2.193968	0.21	2.760061	3.961556	9152.852
0.99149		1.387679	1.381449	0.17	1.96544	4.756177	57039.71
0.98259		1.220758	1.102427	0.15	1.651695	5.069923	117468.8

Table 5.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils $R(CS)$*	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	3.961556	9153
0.99149	0.95	4.756177	57040
0.98259	0.90	5.069923	117469

5. Results for 75% Nominal Operation Stresses

Table 5.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	7.131174	0.638327	5.300432	5.619127	5.81312
2	7.047609	0.643974	5.252974	5.55184	5.726526
3	6.932485	0.630615	5.238439	5.536091	5.694587
4	6.694218	0.613072	5.095603	5.371243	5.527464
5	6.327895	0.534284	4.964344	5.171093	5.304337
Average	6.826676	0.612054	5.170358	5.449879	5.613207

Table 5.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty u_c*
0.99833	1.656318
0.99149	1.376797
0.98259	1.213469

Table 5.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty u_s*
0.99833	6.250682	0.323672	2.216271
0.99149	3.93579		1.395492
0.98259	3.140848		1.113634

degree of freedom =4

Table 5.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17
0.98259	2.11	0.15

Std ~0.07

Table 5.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.8266762	1.656318	2.216271	0.21	2.774769	4.051907	11269.56
0.99149		1.376797	1.395492	0.17	1.967706	4.85897	72271.94
0.98259		1.213469	1.113634	0.15	1.65384	5.172836	148879.8

Table 5.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.051907	11270
0.99149	0.95	4.85897	72272
0.98259	0.90	5.172836	148880

6. Results for 70% Nominal Operation Stresses

Table 5.4 Results of log(life) based on multiple specimen data for crack growth

Specimen N0.	*mean*	*std*	*R= 0.99833*	*R=0.99149*	*R=0.98259*
1	7.240519	0.615388	5.449761	5.751222	5.943903
2	7.156288	0.620337	5.395773	5.692082	5.864157
3	7.042161	0.615423	5.37212	5.646908	5.817408
4	6.806593	0.606235	5.209348	5.499487	5.64233
5	6.429724	0.53444	5.057935	5.2568	5.404195
Average	6.935057	0.598365	5.296987	5.5693	5.734399

Table 5.5 Uncertainty due to crack growth based on the evaluation of mean over all crack growth data at the selected reliabilities

Reliability R	*Uncertainty* u_c
0.99833	1.63807
0.99149	1.365757
0.98259	1.200658

Table 5.6 Uncertainty due to small sample statistics by student t distribution

Reliability R	*t*	*Std of mean-log(life)*	*Uncertainty* u_s
0.99833	6.250682	0.326184724	2.233478
0.99149	3.93579		1.406326
0.98259	3.140848		1.12228

degree of freedom =4

Table 5.7 Uncertainty due to minor factors

Reliability R	*Scale factor*	*Estimated uncertainty* u_m
0.99833	2.94	0.21
0.99149	2.39	0.17
0.98259	2.11	0.15

Std ~0.07

Table 5.8 Fatigue life estimation for one CS coil

R	*Mean*	*Uncertainty* u_c	*Uncertainty* u_s	*Uncertainty* u_m	*Total* u_{tot}	*Log(life)*	*Life (cycle)*
0.99833	6.935057	1.63807	2.233478	0.21	2.777732	4.157325	14365.64
0.99149		1.365757	1.406326	0.17	1.967726	4.967331	92753.58
0.98259		1.200658	1.12228	0.15	1.650331	5.284726	192630.9

Table 5.9 Fatigue life estimation for 6 CS coils

Reliability for 1 coil $R(coil)$	*Reliability for 6 coils* $R(CS)$	*Log(life)*	*Life (stress cycle)*
0.99833	0.99	4.157325	14366
0.99149	0.95	4.967331	92754
0.98259	0.90	5.284726	192631